Engines of Parsimony

computing on a budget

Hannah Earley

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physics of computing

computational performance constraints

dissipation in reversible computing (RC)

super-adiabatic RC?

parallelism & concurrency in RC

Thermodynamics of Computing

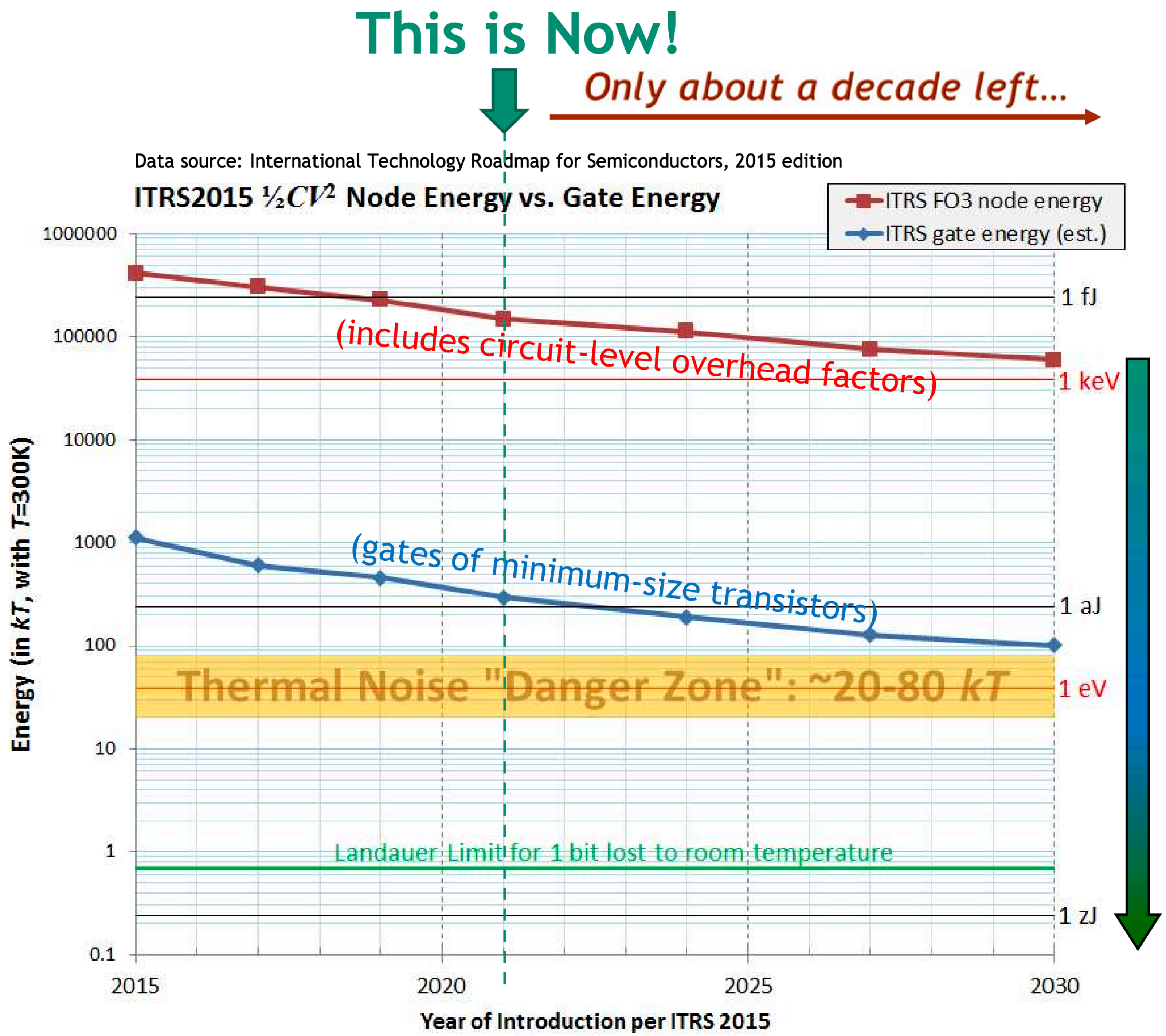
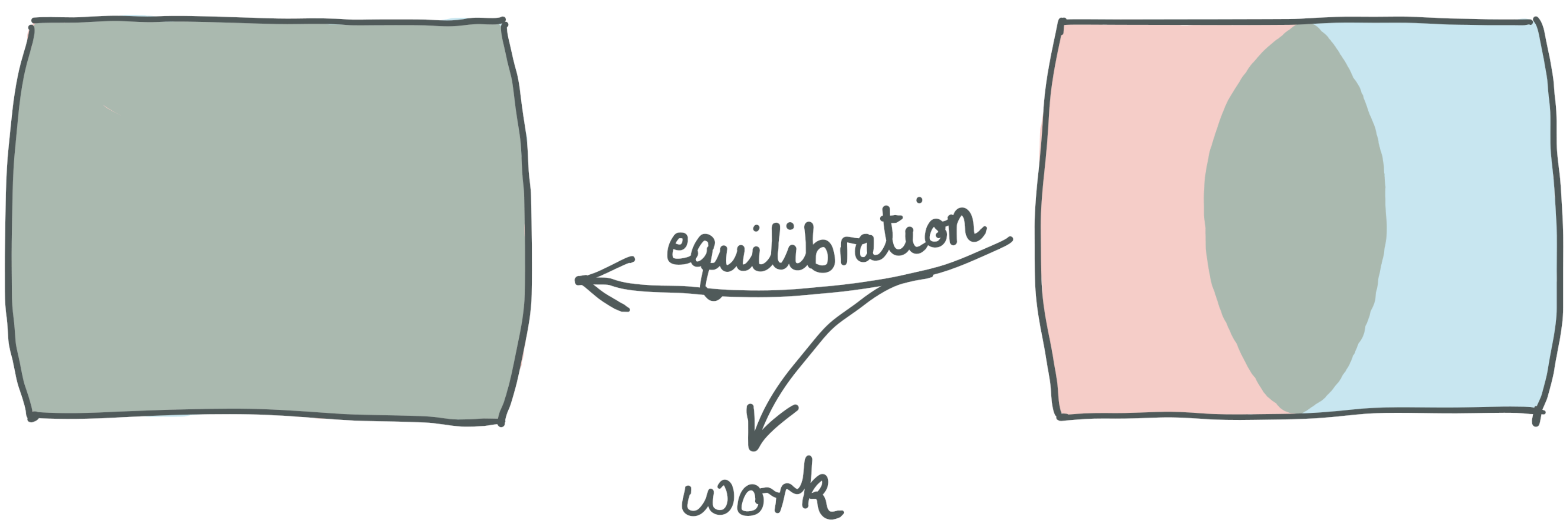


Figure — Mike Frank

Maxwell's Daemon



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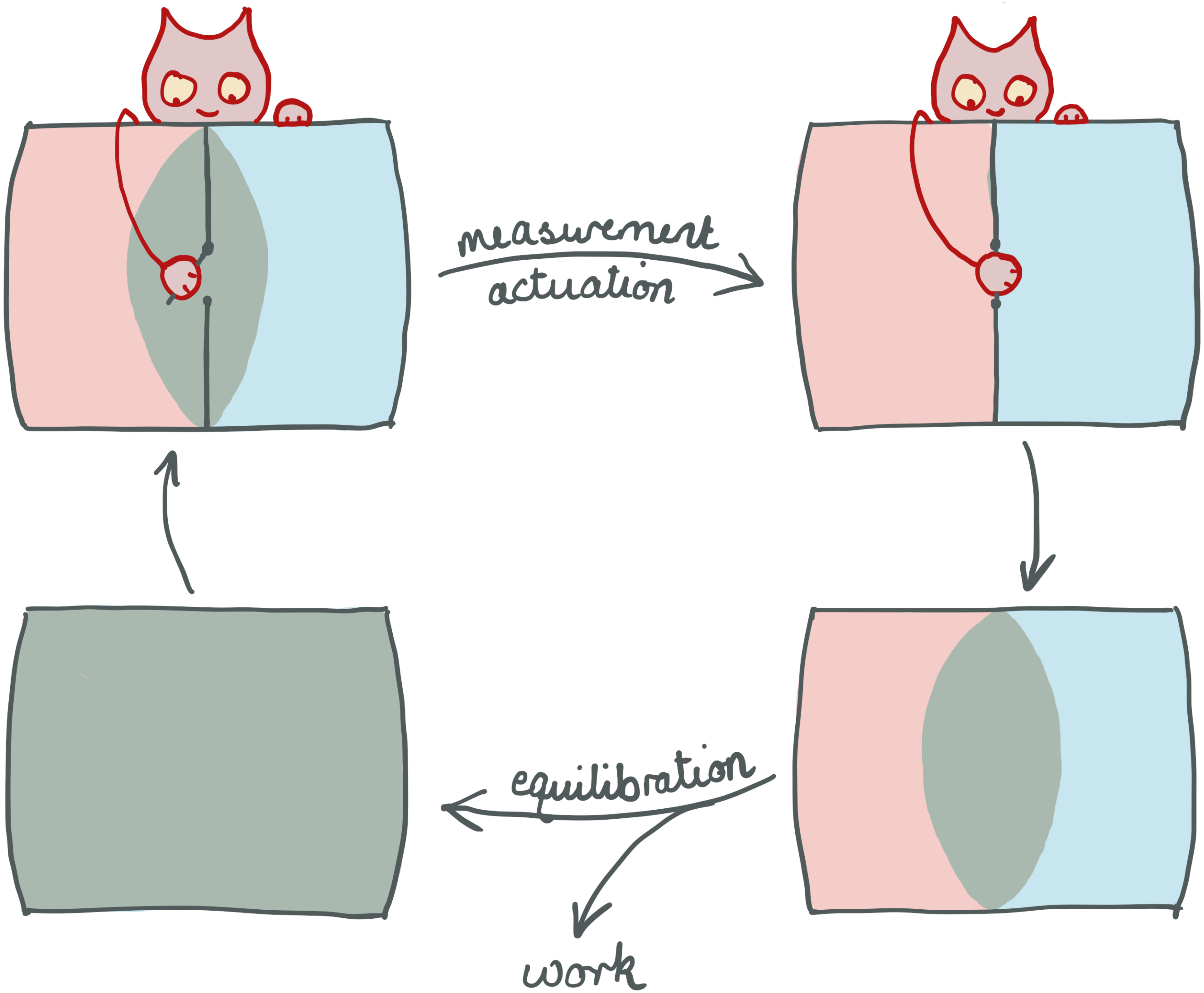
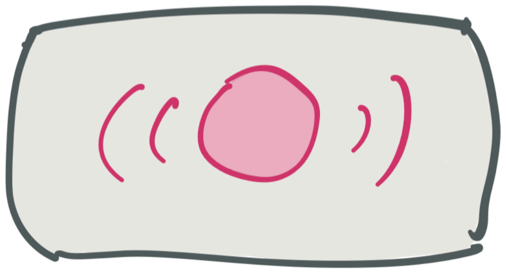


Szilard Engine



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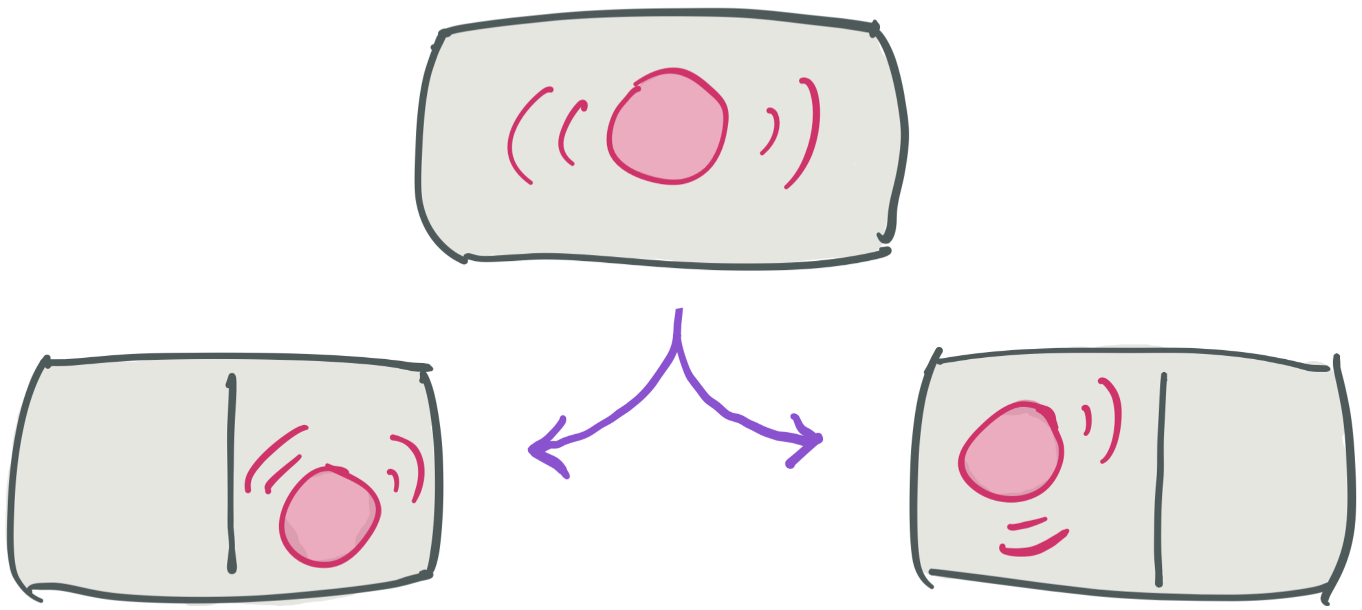

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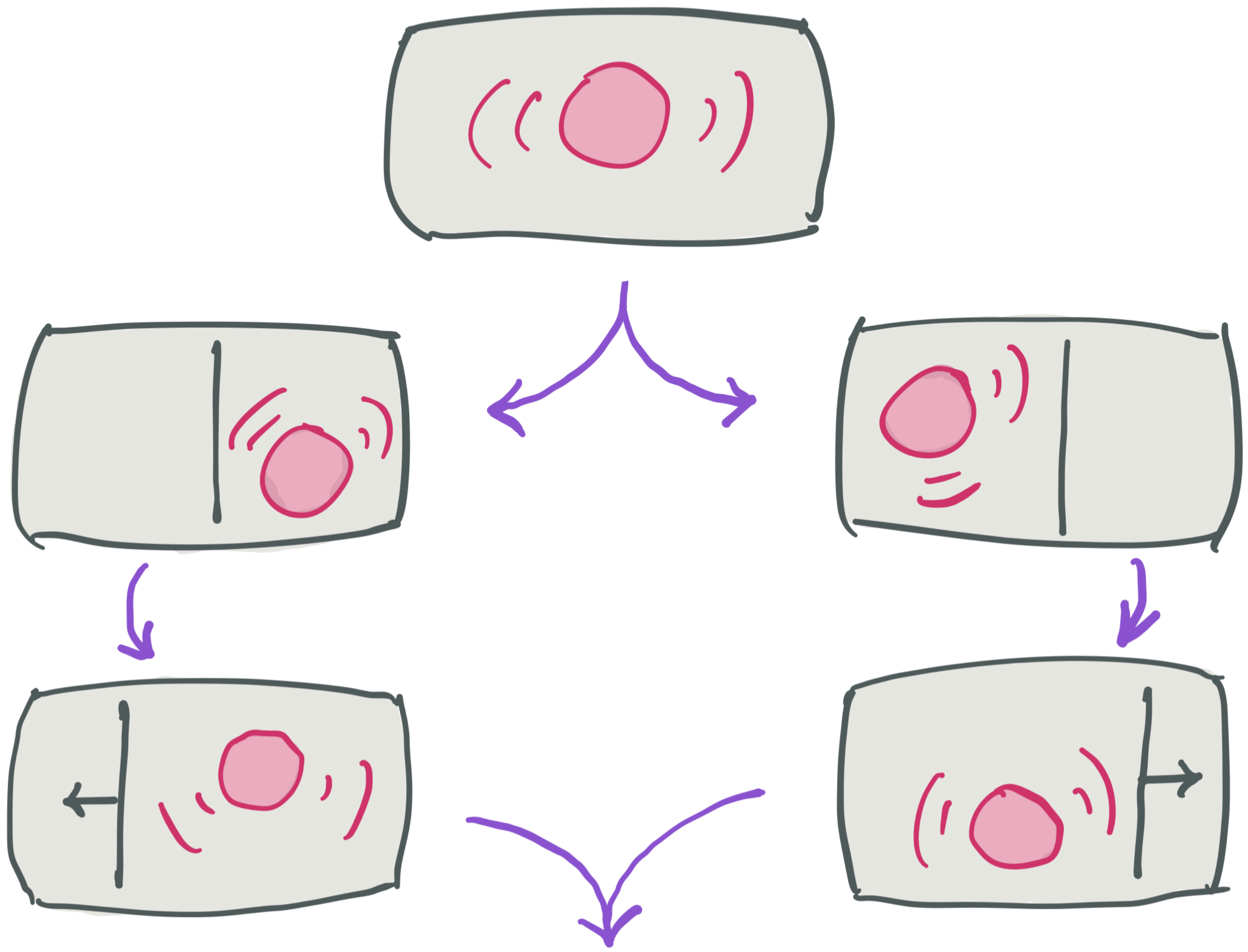

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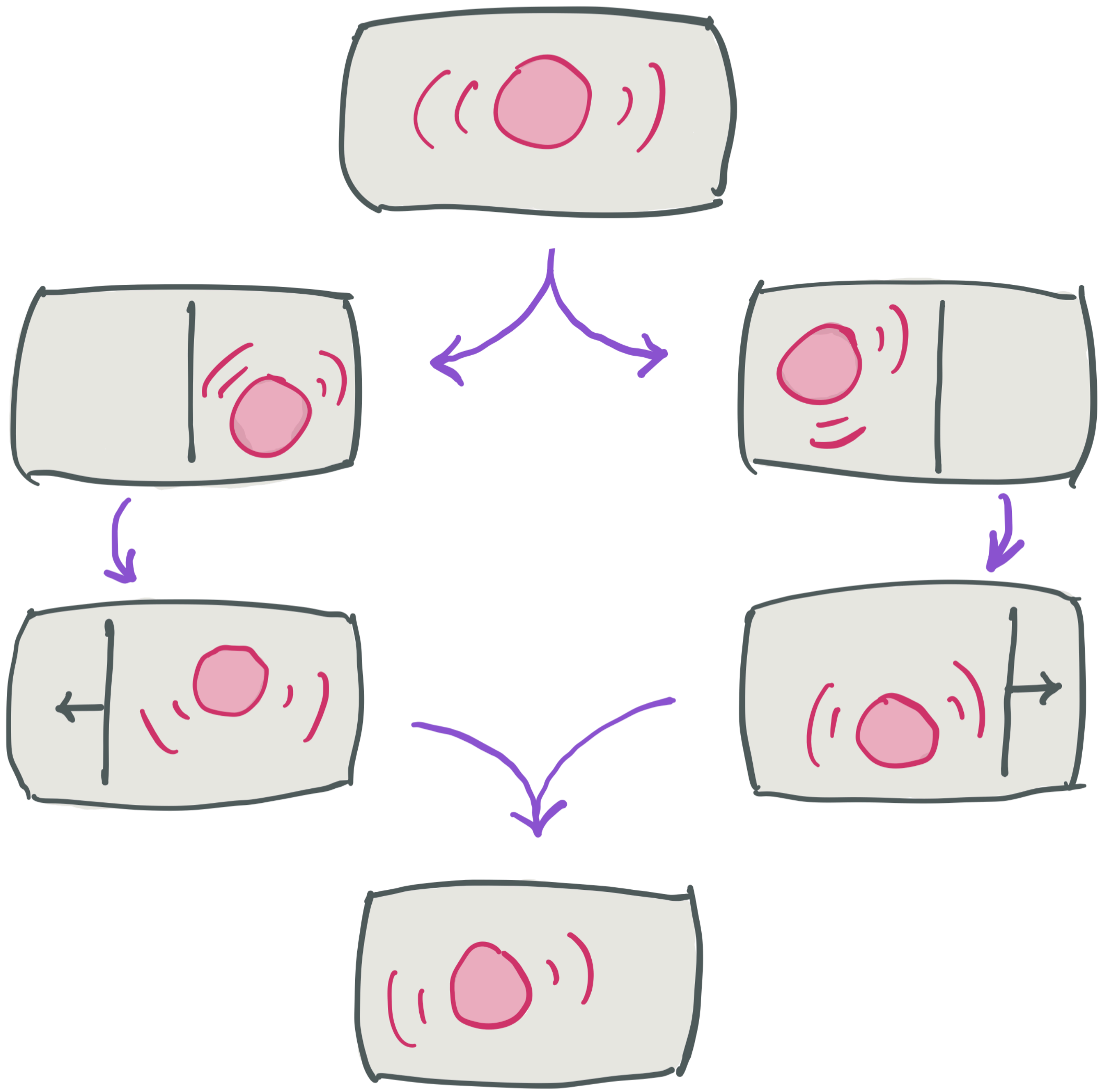

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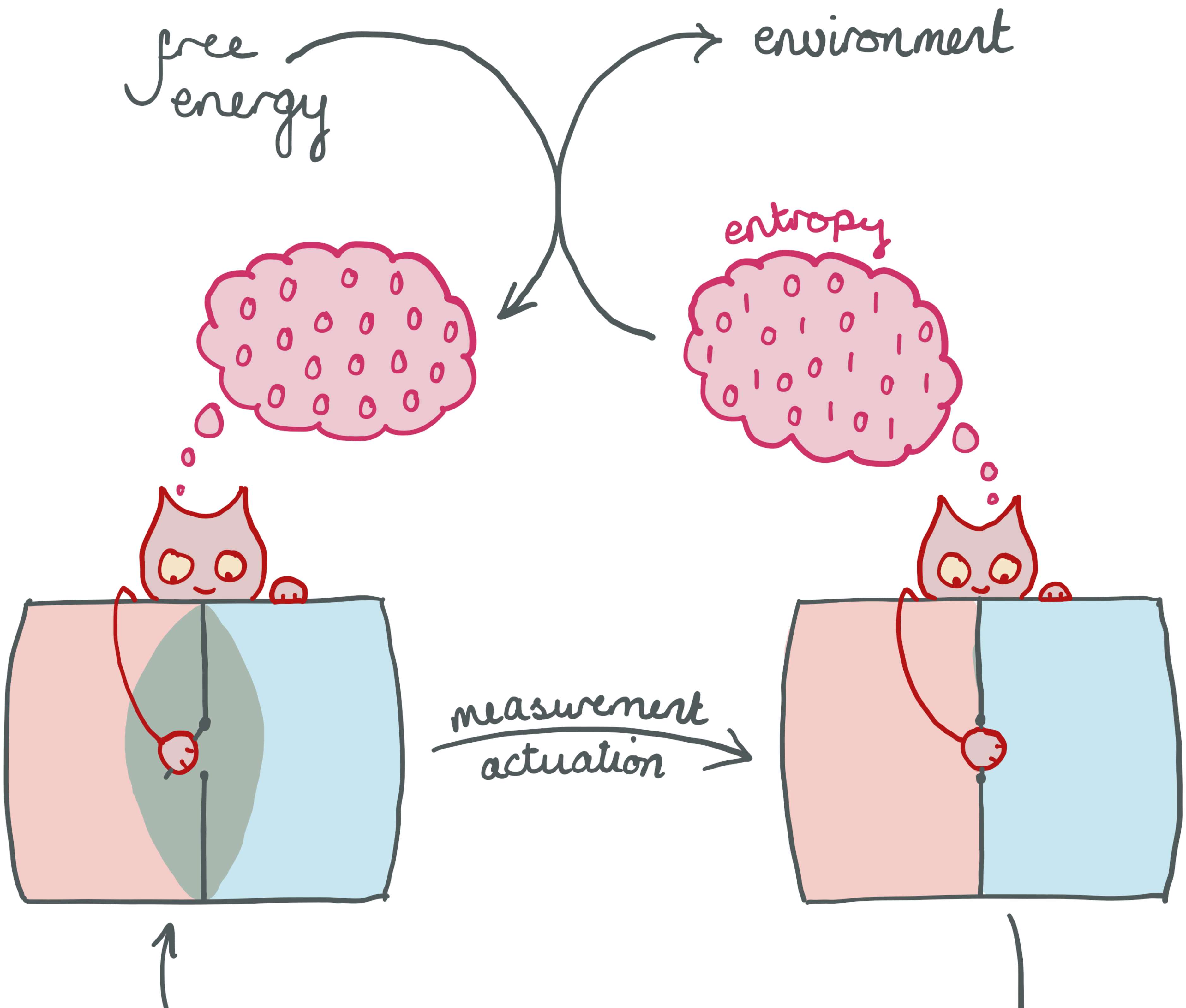

Landauer Limit

$$\Delta S \geq k_B \Delta I$$

$$\Delta Q \geq k_B T \log 2$$

Landauer, Rolf. "Irreversibility and heat generation in the computing process." IBM journal of research and development 5.3 (1961): 183-191.
Frank, Michael P., and Karpur Shukla. "Quantum foundations of classical reversible computing." Entropy 23.6 (2021): 701.

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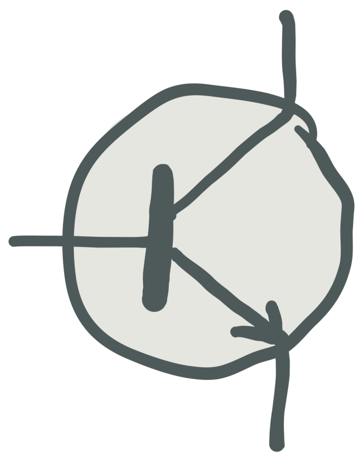
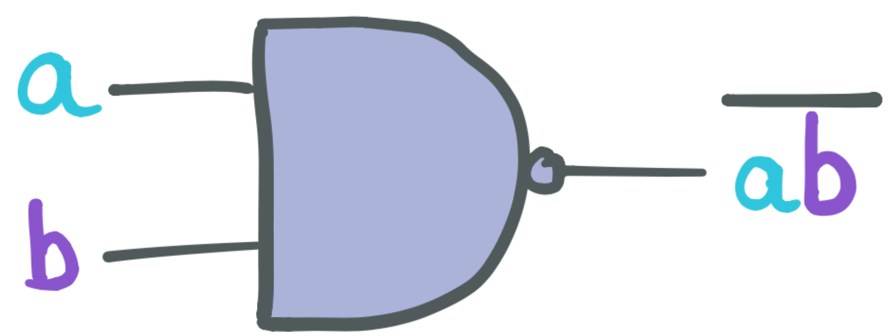
Consequences for Computing



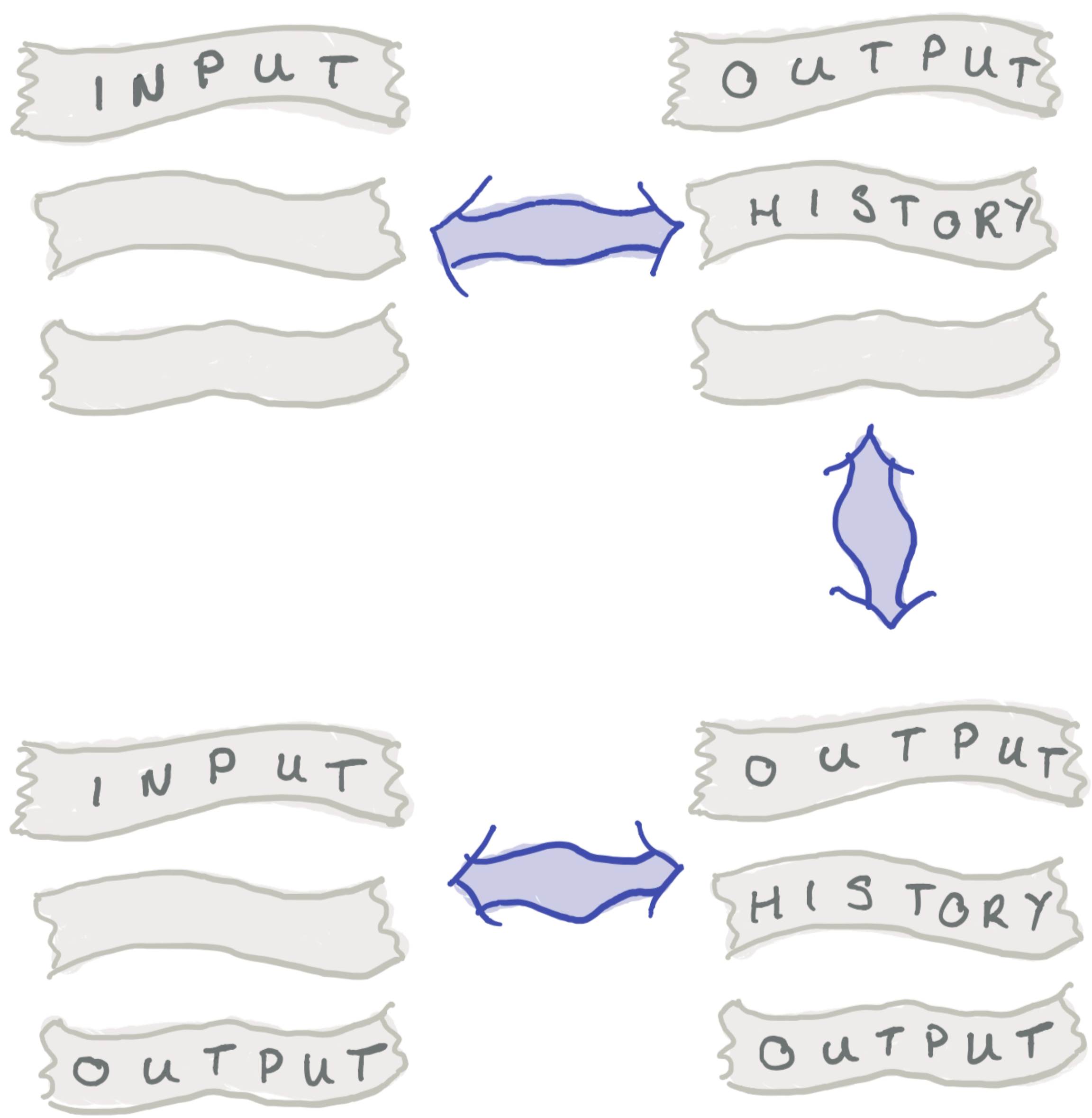



```
mov AL, 314
jne loop
```

```
void swap(int *a, int *b) {
  int tmp = *b;
  *b = *a;
  *a = tmp;
}
```

```
int i = 5;
while (i--) {
  printf("%d\n", i);
}
```

Reversible Computing



$n\ x[2]$

procedure *fib*

  if $n = 0$

  then $x_0 \mathrel{+}= 1$

    $x_1 \mathrel{+}= 1$

  else $n \mathrel{-}= 1$

    call *fib*

    $x_0 \mathrel{+}= x_1$

    $x_0 \Leftrightarrow x_1$

  fi $x_0 = x_1$

Bennett, Charles H. "Logical reversibility of computation." IBM journal of Research and Development 17.6 (1973): 525-532.
Yokoyama, Tetsuo, and Robert Glück. "A reversible programming language and its invertible self-interpreter." Proceedings of the 2007 ACM SIGPLAN symposium on Partial evaluation and semantics-based program manipulation. 2007.

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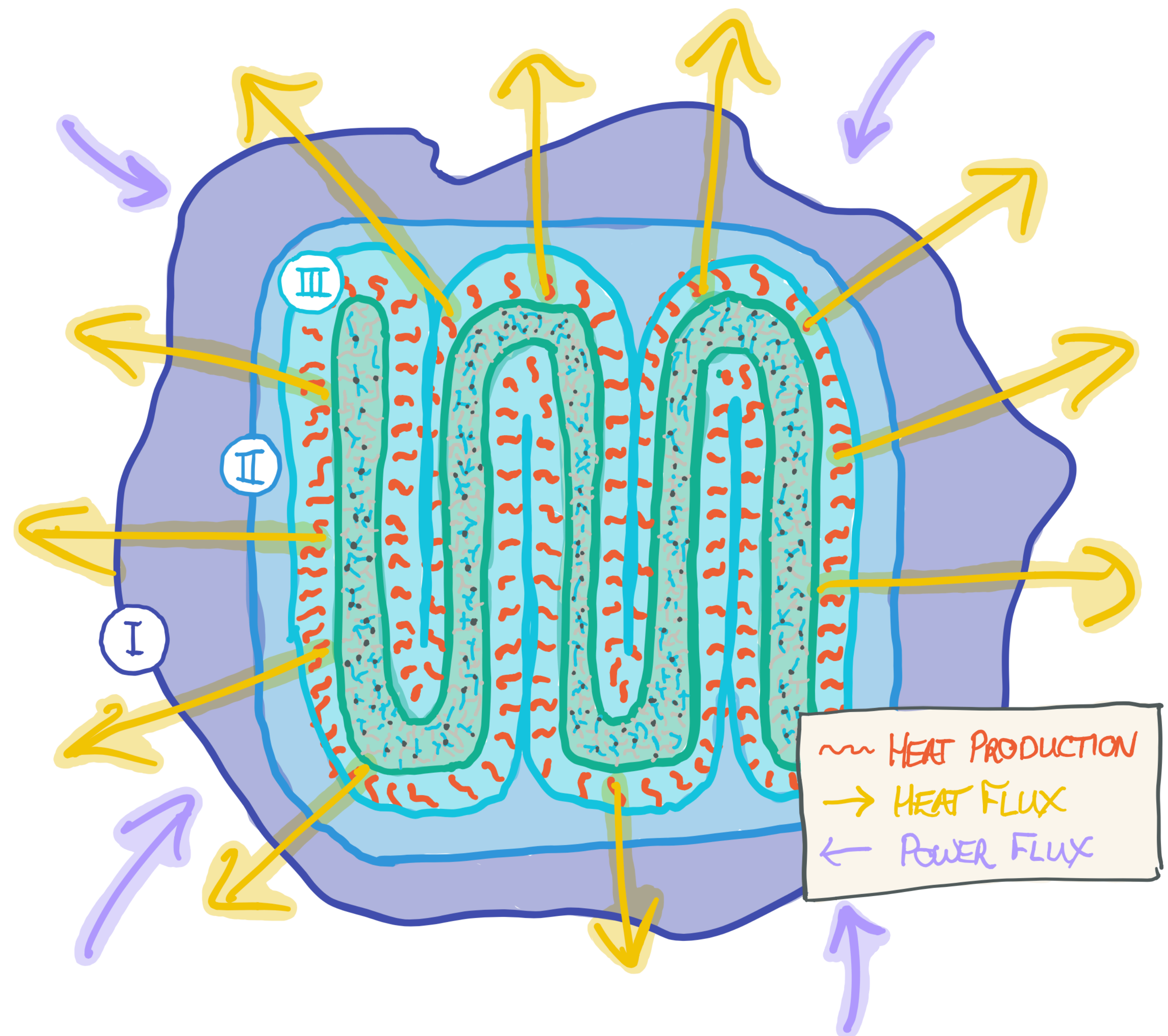
parallelism & concurrency in RC

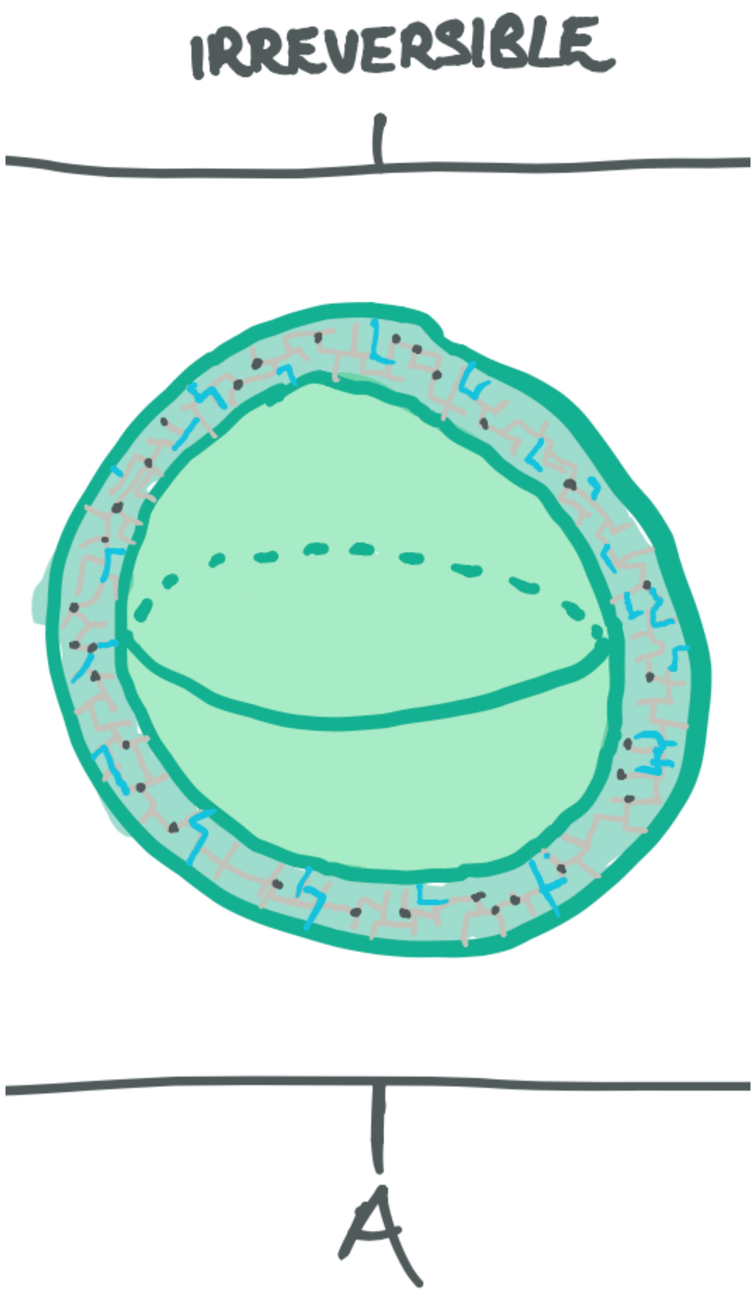
Setup

- mass-energy available for computation
  - prop to volume
- heat/free energy flux
  - prop to convex boundary



Conventional Computers

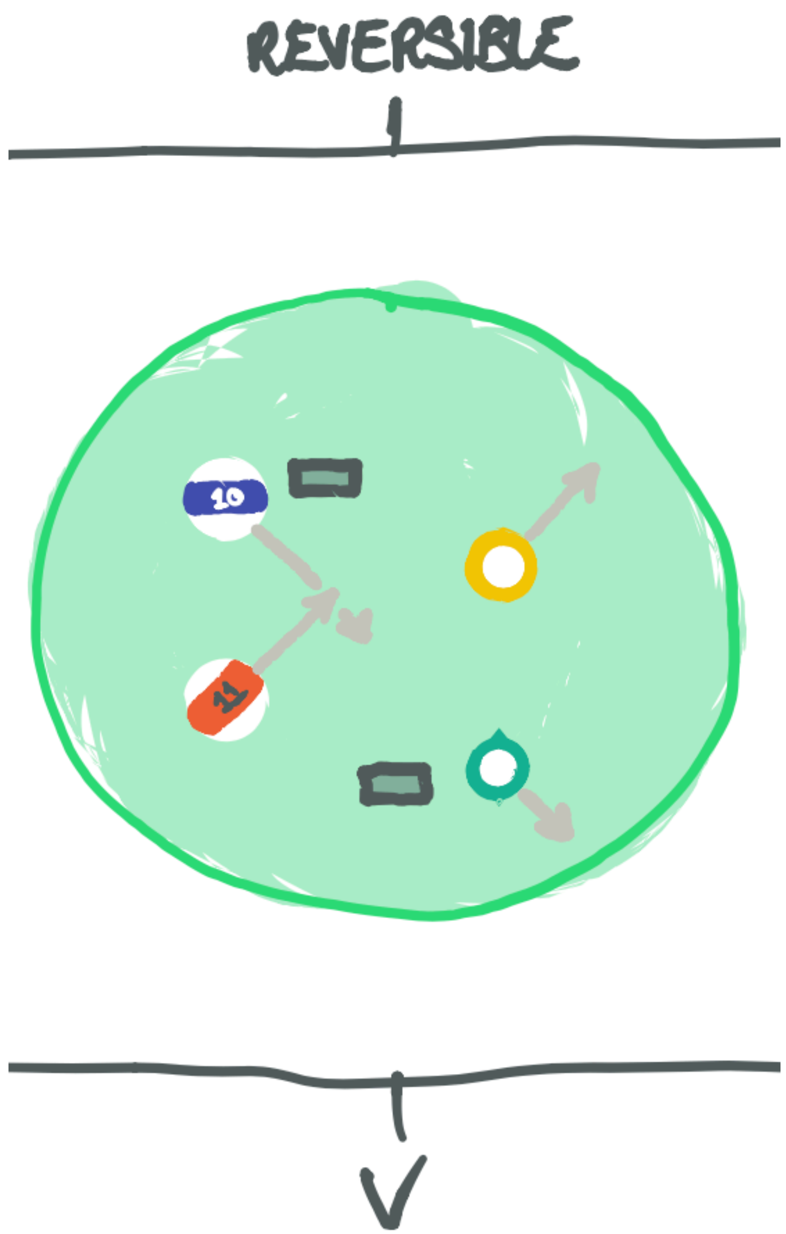
- Landauer Limit lower bound
- P/A constrained



Ballistic Computing

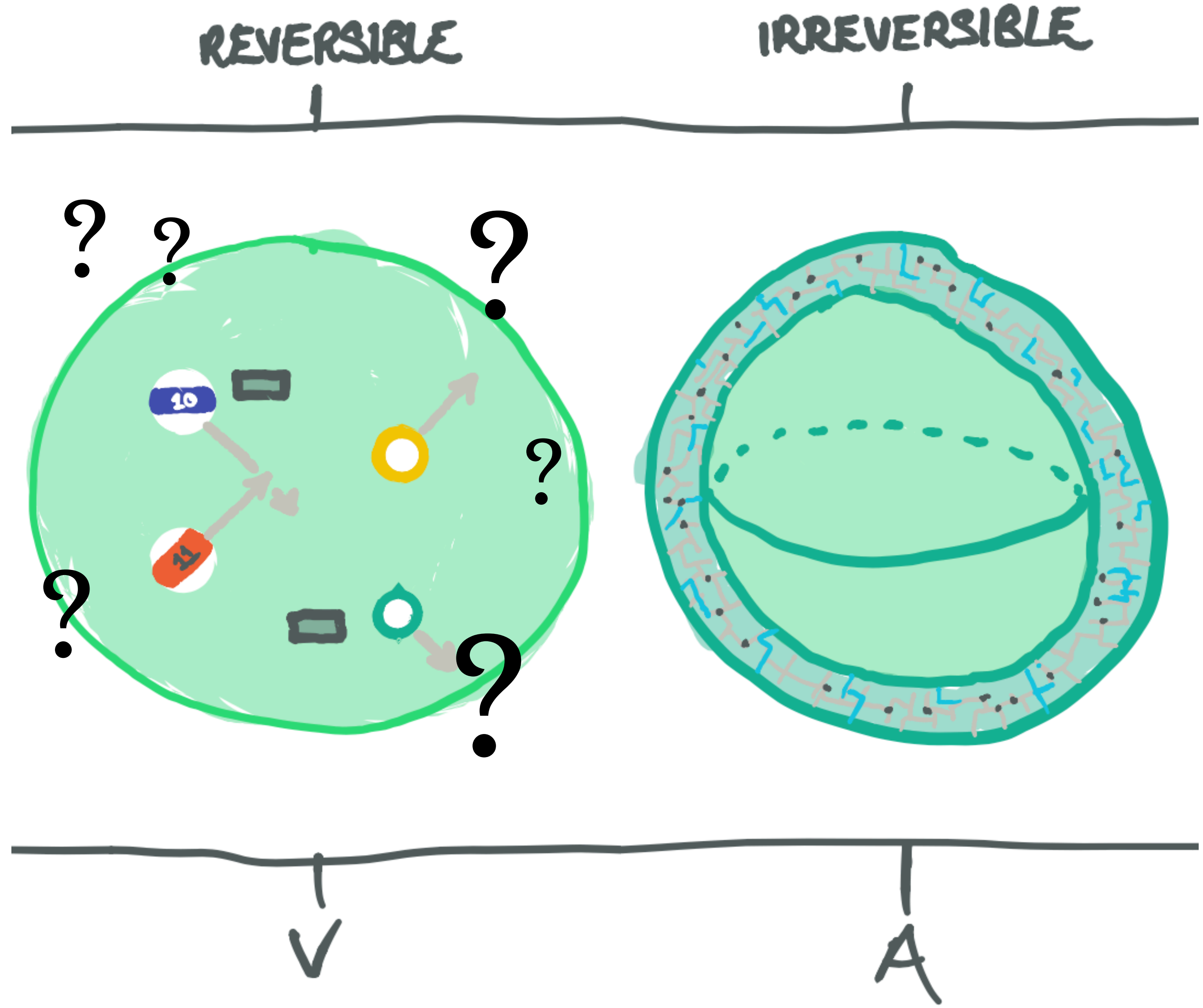
- RC not subject to Landauer limit
- Computing for free(...?)
- Not P-limited...
- ...but M-limited

$$\nu \leq 2E/h$$
$$\leq E/\pi$$



Margolus, Norman, and Lev B. Levitin. "The maximum speed of dynamical evolution." Physica D: Nonlinear Phenomena 120.1-2 (1998): 188-195.

architecture



net rate of computation

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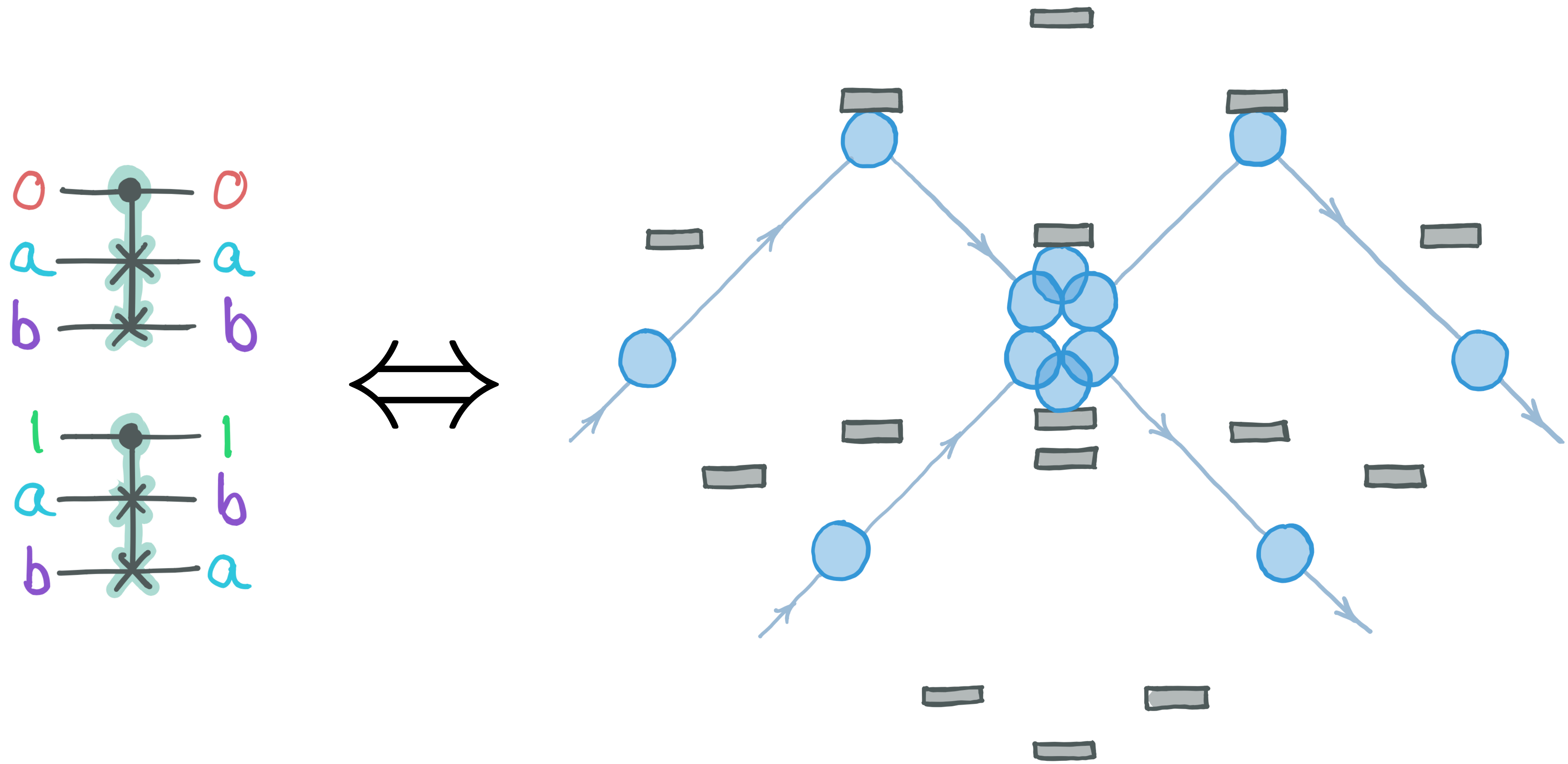
Ballistic Computing?



Fredkin, Edward, and Tommaso Toffoli. "Conservative logic." International Journal of theoretical physics 21.3 (1982): 219-253.

Ballistic Computing?

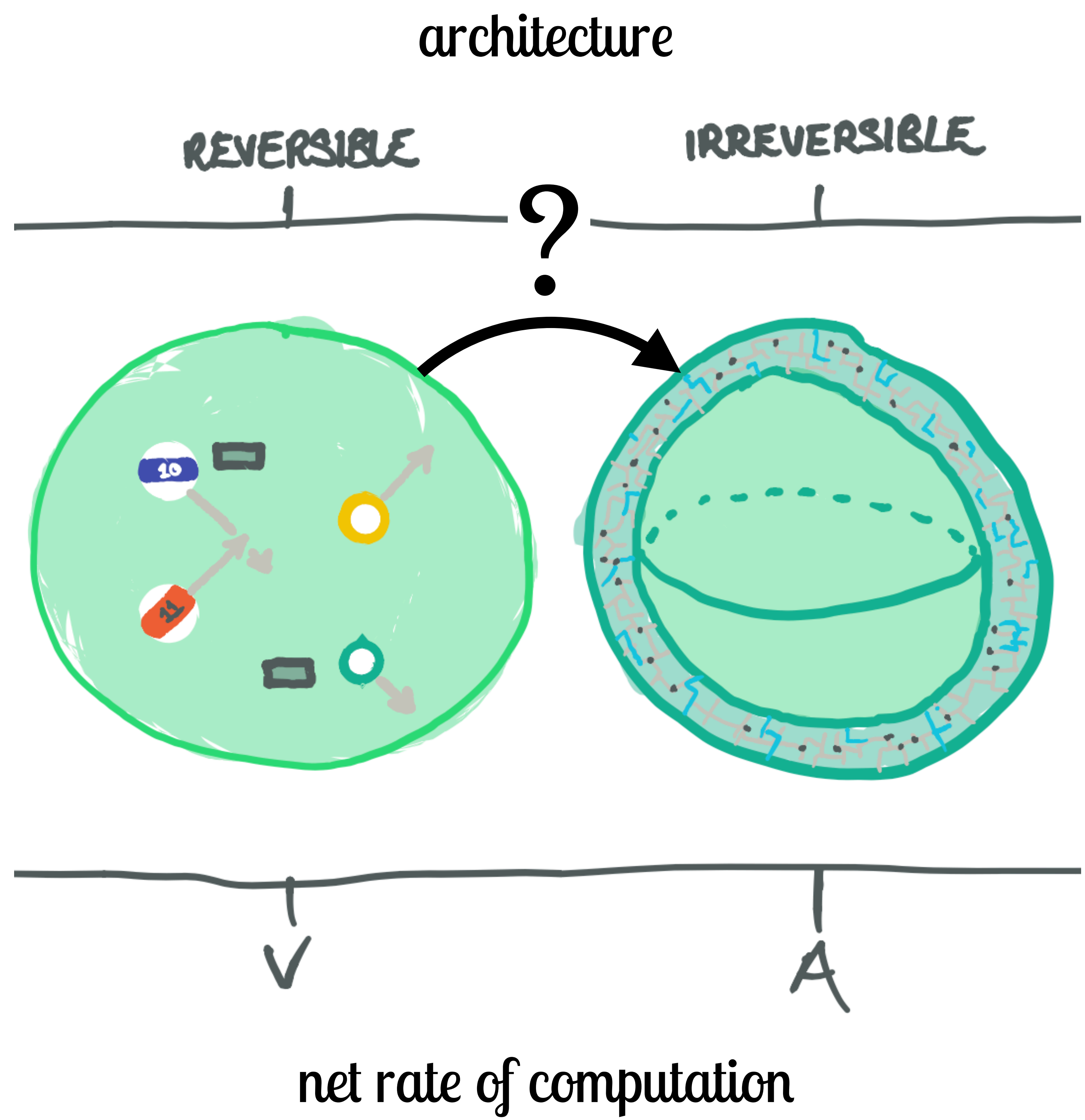
"Even if classical balls could be shot with perfect accuracy into a perfect apparatus, fluctuating tidal forces from turbulence in the atmospheres of nearby stars would be enough to randomize their motion within a few hundred collisions. Needless to say, the trajectory would be spoiled much sooner if stronger nearby noise sources (e.g., thermal radiation and conduction) were not eliminated."

-Charles Bennett

Bennett, Charles H. "The thermodynamics of computation—a review." International Journal of Theoretical Physics 21.12 (1982): 905-940.

Dissipation in RC

- thermo coupling
- signal/state restoration
- superconductivity? only DC...



Adiabatic RC

- asymptotically vanishing (but non-zero) dissipation

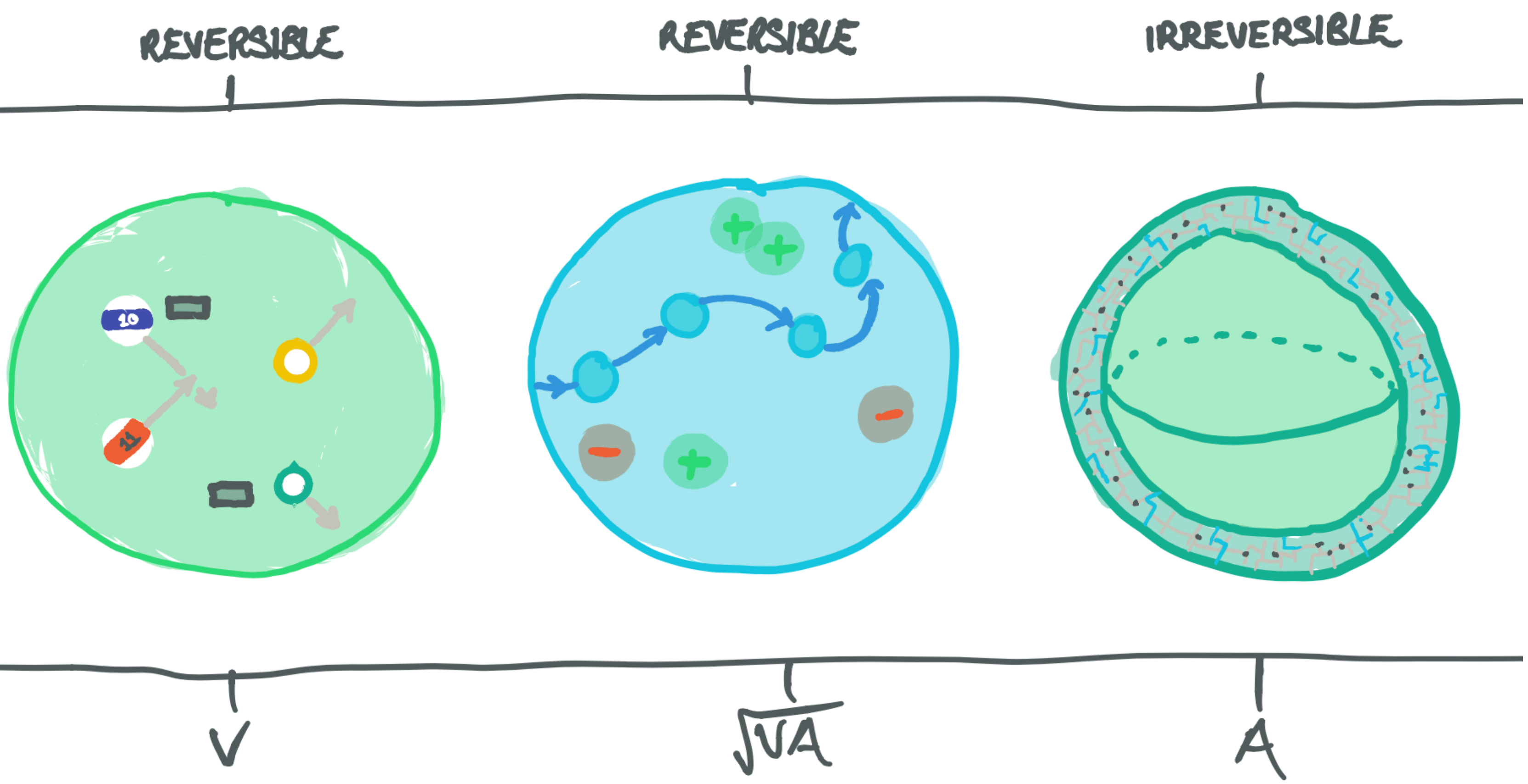
$$\Delta S \cdot \Delta \tau = \text{const.}$$

$$\Delta S \propto \nu$$

- realizable! adiabatic CMOS, ...

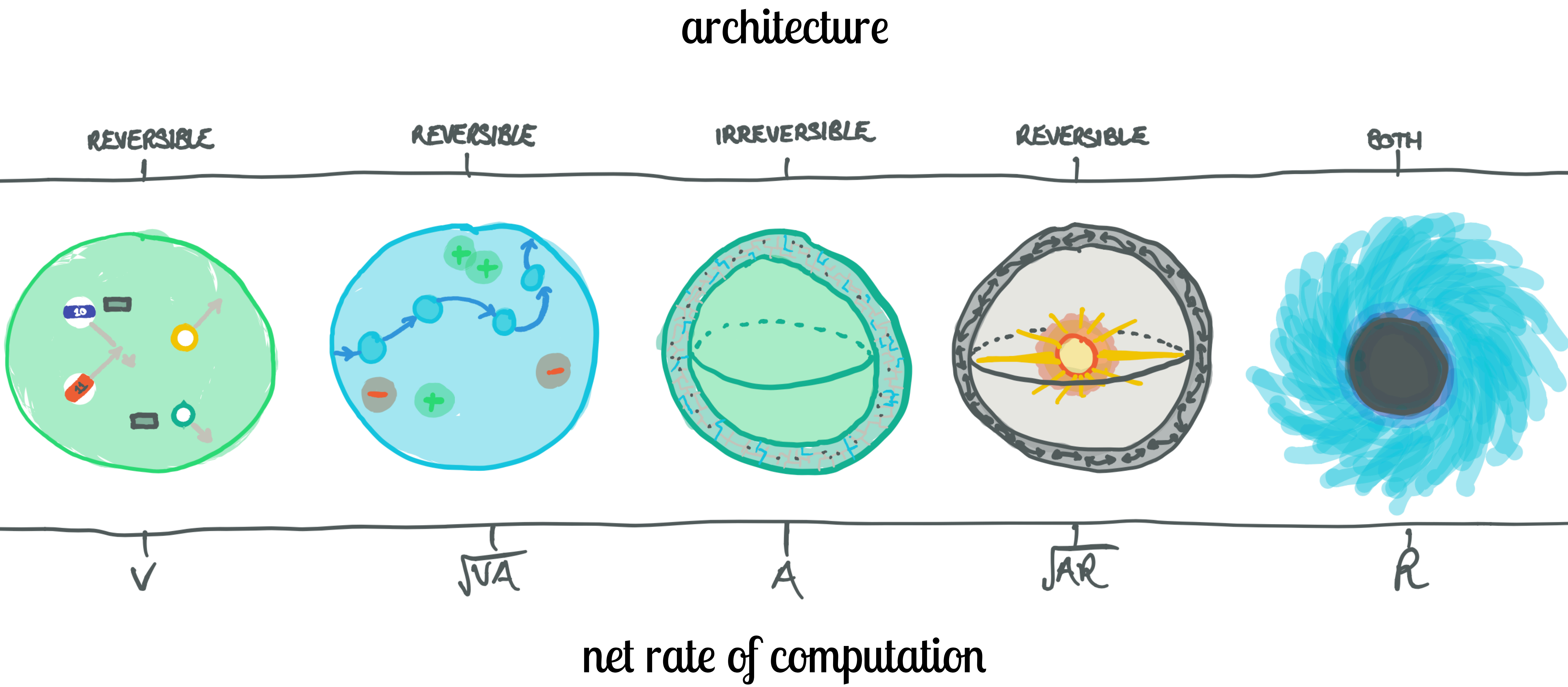
Adiabatic RC

architecture



net rate of computation

Adiabatic RC



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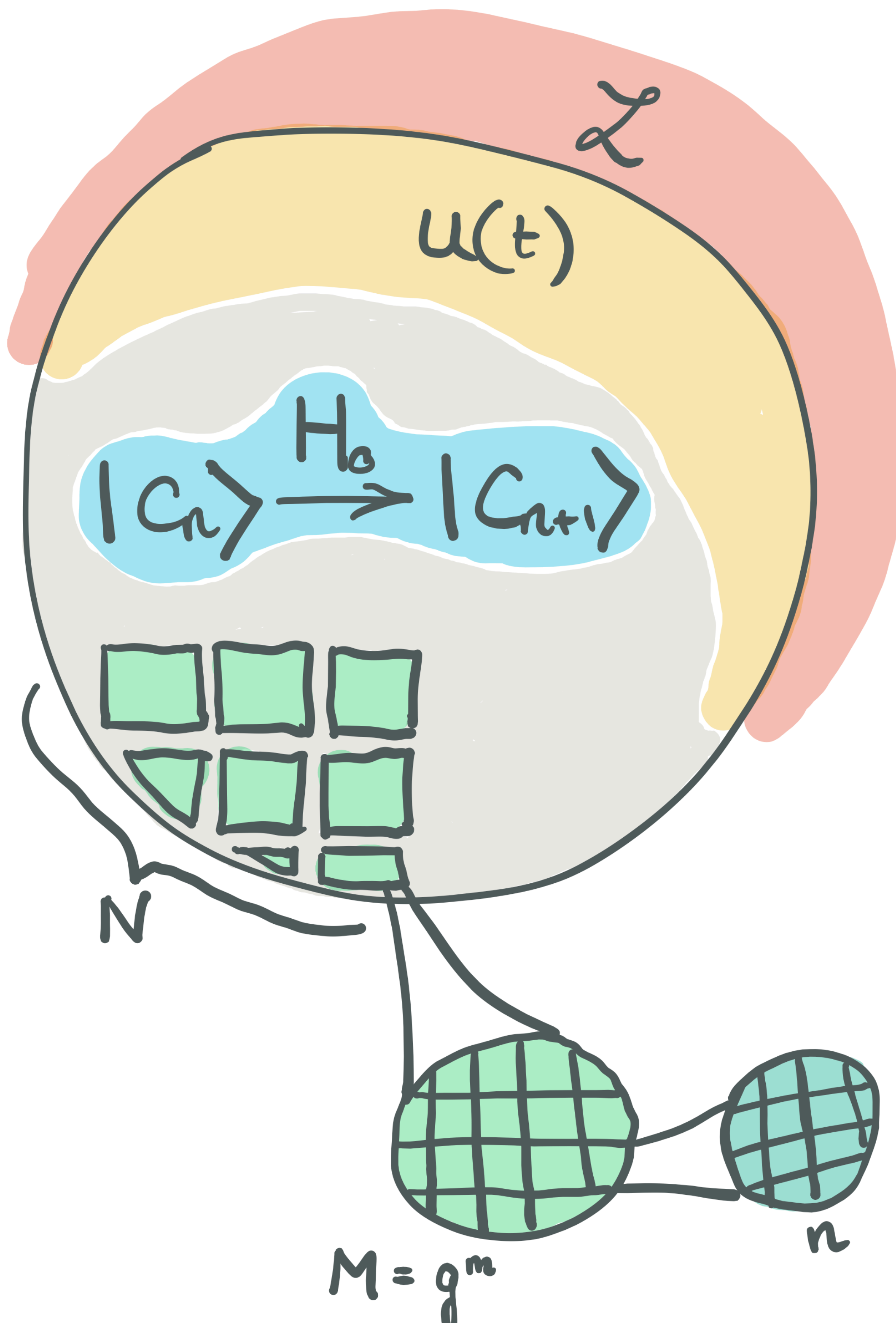
Generic RC Model

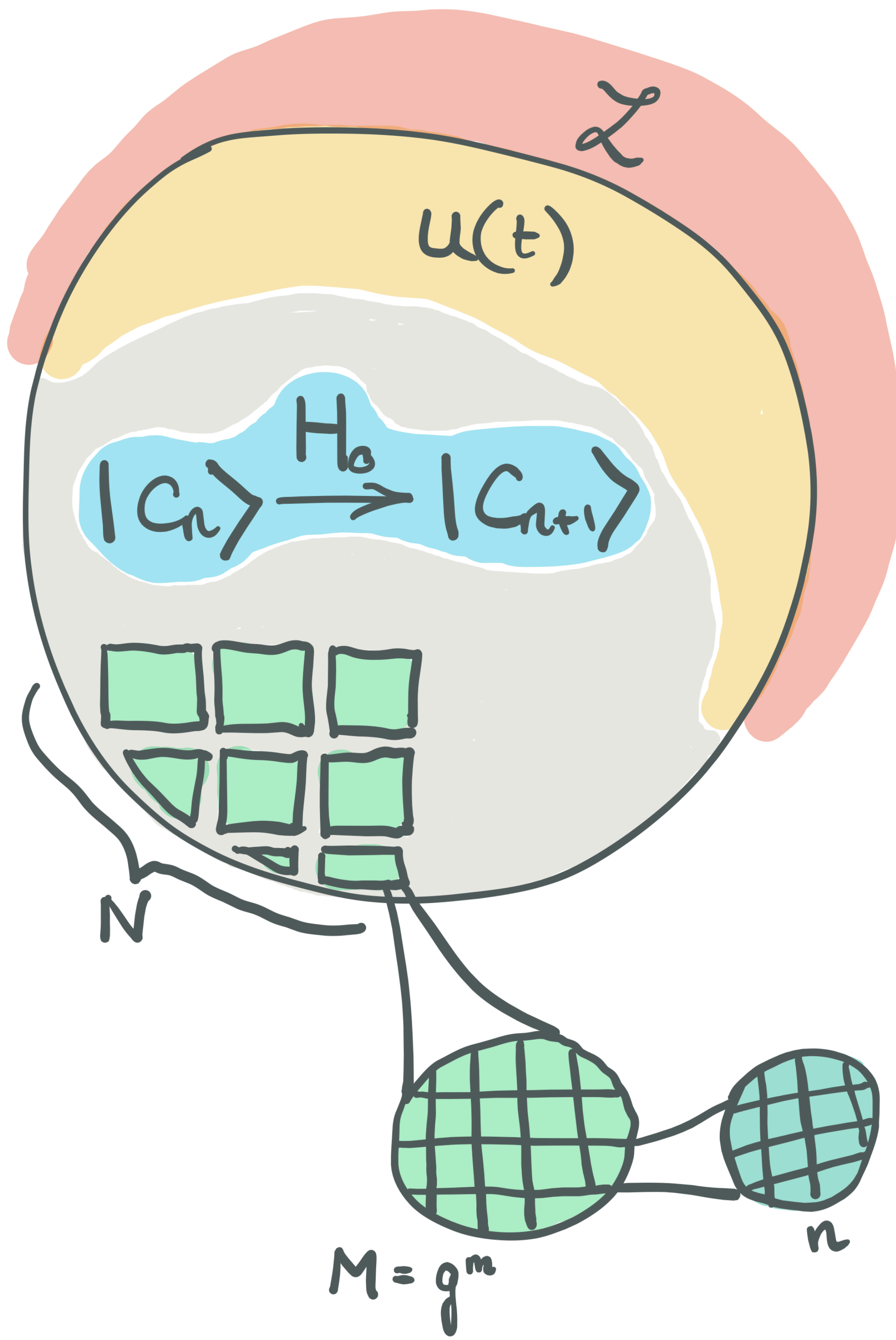


$$H = H_0 + U(t)$$

$$\dot{\rho} = i[\rho, H] + \mathcal{L}\rho$$

$$\mathcal{L}\rho = \sum_i \gamma_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})$$

Generic RC Model



ballistic — internal temp

$$H = H_0 + U(t)$$

$$\dot{\rho} = i[\rho, H] + \mathcal{L}\rho$$

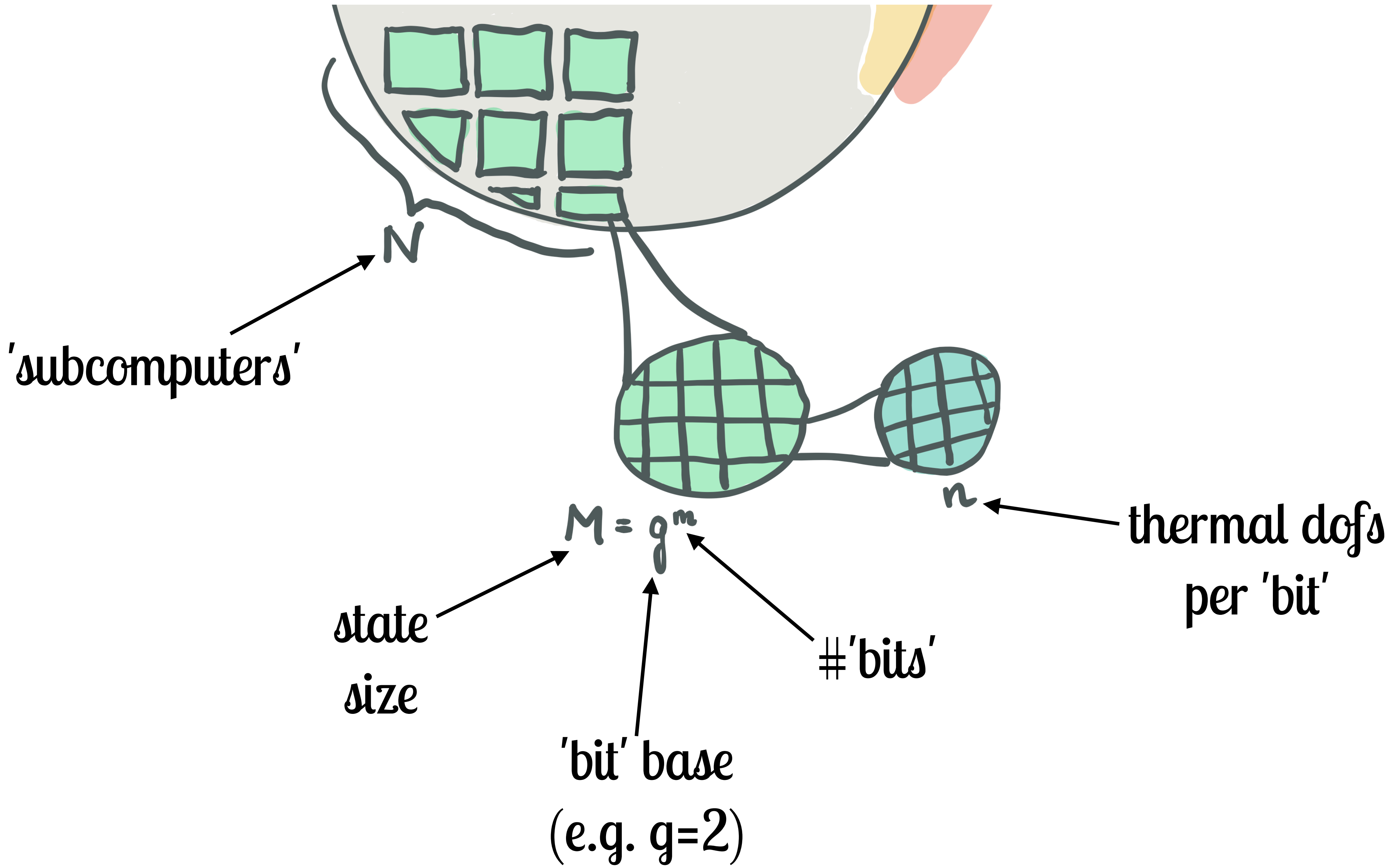
external env

strength

$$\mathcal{L}\rho = \sum_i \gamma_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})$$

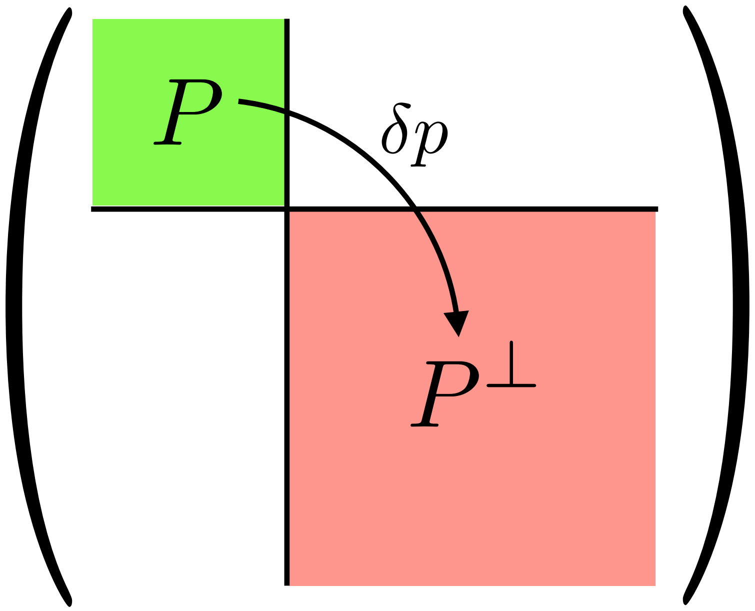
jump ops

Generic RC Model



Error States

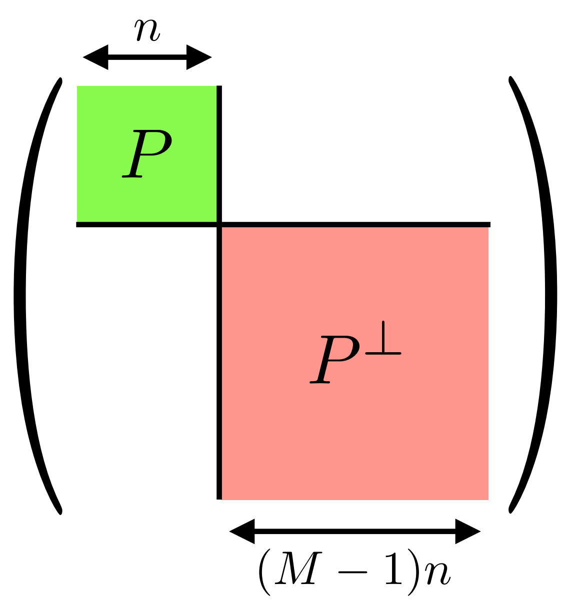
- projectors $P \quad P^{\perp}$
- probability of error δp
  - after small time δt



$$\delta p = \mathrm{tr}[P^{\perp}(\rho + \delta t \dot{\rho} + \tfrac{1}{2}\delta t^2 \ddot{\rho})] + \mathcal{O}(\delta t^3)$$

$$= \mathrm{tr}[P^{\perp}(\delta t \mathcal{L}\rho + \delta t^2 (H \rho H + \tfrac{1}{2} i[\mathcal{L}\rho, H] + \tfrac{1}{2}\tfrac{\mathrm{d}}{\mathrm{d}t}\mathcal{L}\rho))]$$

Entropy Production



- pre-error $\rho = \rho P \qquad S = S_0$
- post-error $S = S_0 + \log(M-1)$
- entropy production

$$\delta\Sigma = [-\delta p \log \delta p - (1-\delta p)\log(1-\delta p) + \delta p(S_0 + \log(M-1)) + (1-\delta p)S_0] - [S_0]$$

$$= \delta p(1 + \log(M-1) - \log(\delta p)) + \mathcal{O}(\delta p^2)$$

$$\geq \delta p \log M$$

Quantum Zeno RC

- weak Lindbladian $\gamma_i \lesssim \mathcal{O}(\delta t)$

- error probability

$$\begin{aligned}\delta p &= \delta t^2 \operatorname{tr}[\rho(HP^\perp H + \textstyle\sum_i (\gamma_i/\delta t) L_i^\dagger P^\perp L_i)] + \mathcal{O}(\delta t^3) \\ &= \delta t^2 \operatorname{tr}[\rho_{00}(U_{01}U_{01}^\dagger + \textstyle\sum_i (\gamma_i/\delta t) L_{i,01}^\dagger L_{i,01})] \\ &= \delta t^2 \langle U' U'^\dagger \rangle\end{aligned}$$

effective thermal potential

- QZE $\delta p \sim \mathcal{O}(\delta t^2)$

Quantum Zeno RC

- entropy production rate

$$\dot{\Sigma} \geq \delta t \left\langle U' U'^{\dagger} \right\rangle m \log g$$

- effective potential couples to each thermal dof nm
  - coupling uncorrelated $\left\langle U' U'^{\dagger} \right\rangle \approx nmT^2$ ← effective temperature

$$\dot{\Sigma} \geq \delta t \, nm^2 T^2 \log g$$

Quantum Zeno RC

- Zeno and computation rates (M–L)

$$\nu_Z = 1/\delta t \qquad\qquad \nu_C \leq E_C/\pi n$$
$$\leq E_Z/\pi \qquad\qquad = nm\varepsilon/\pi n$$

- entropy generation rate // computation rate

$$\dot{\Sigma} \geq \frac{\nu_C^2}{\nu_Z} \underbrace{\frac{n\pi^2 T^2}{\varepsilon^2} \log g}_{\eta}$$

$(\Delta\Sigma = \dot{\Sigma}/\nu_C)$

- adiabatic bound!

$$\Delta\Sigma = \nu_C(\eta/\nu_Z) \propto \nu_C \qquad\qquad \nu_c \leq \sqrt{\frac{1}{\pi\eta}\dot{\Sigma}E}$$

Strong Lindbladian RC

- strong Lindbladian $\gamma_i \sim \mathcal{O}(1)$
- lose QZE $\delta p = \delta t \, \mathrm{tr}[\rho \sum_i \gamma_i L_i^\dagger P^\perp L_i]$
- retain adiabaticity $\dot{\Sigma} \geq n m^2 T^2 \log g = \eta \nu_C^2$
    - BUT lose geometric scaling?? $\nu_C \lesssim \sqrt{\dot{\Sigma}} \sim r$

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- $\Rightarrow$ subdivide $\nu_C \lesssim \sqrt{N \dot{\Sigma}} \sim r^{5/2}$
  - (individually slow) $\nu_{C,1} \lesssim r^{-1/2}$

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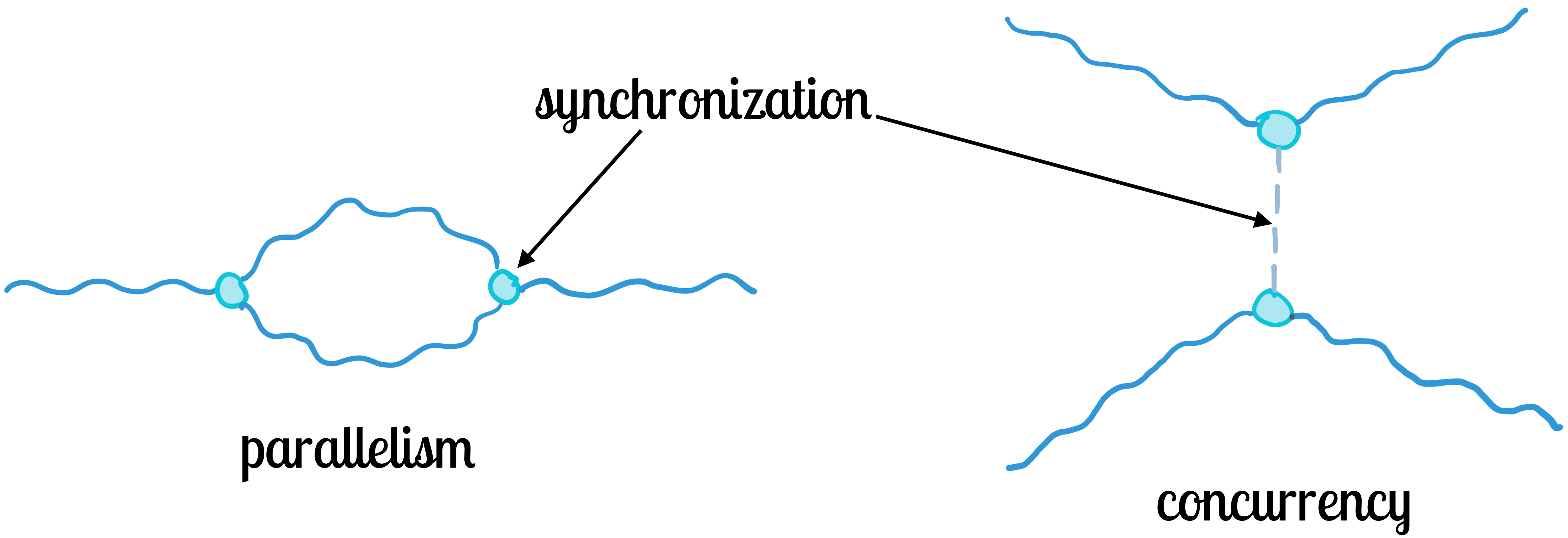
Synchronization



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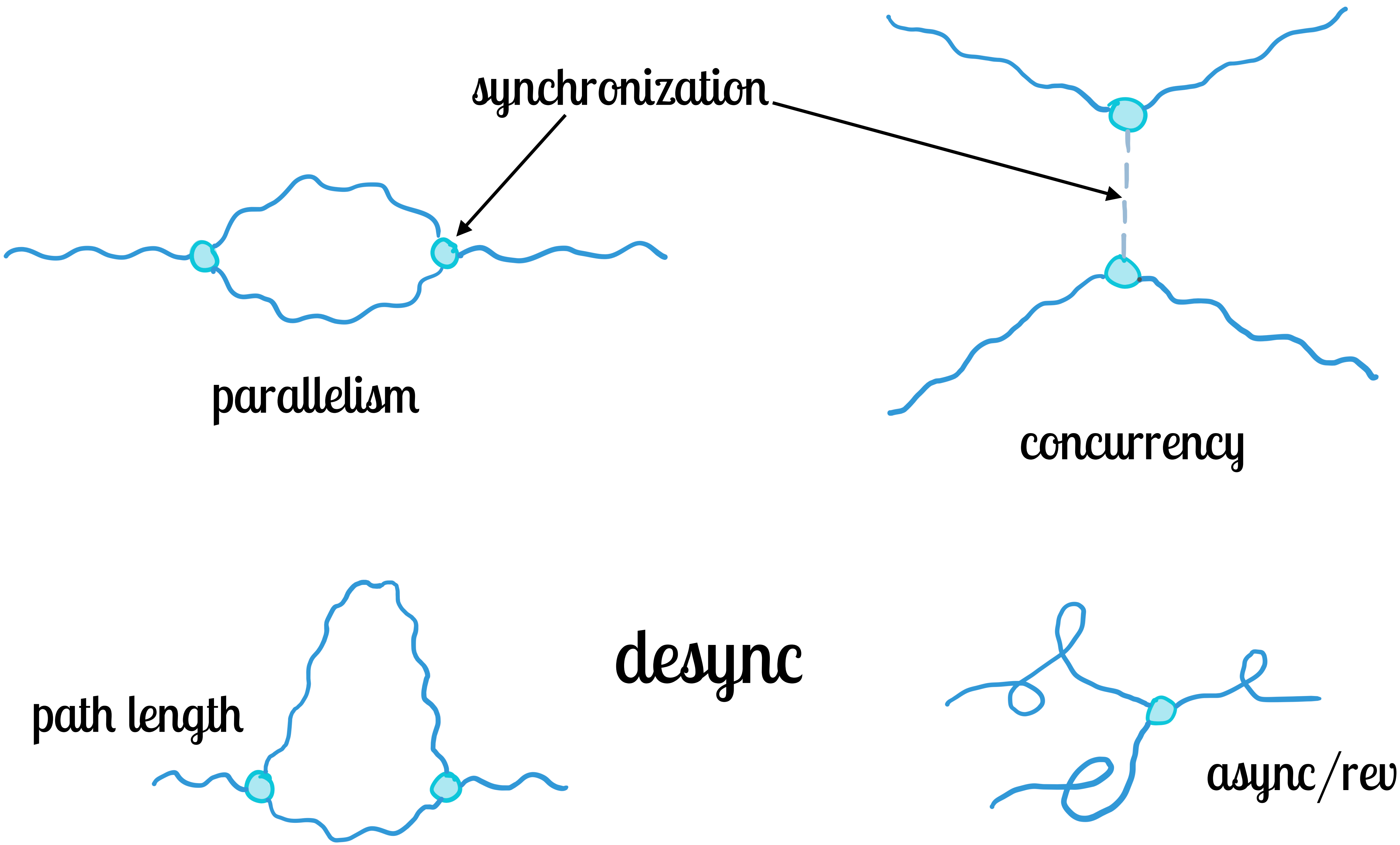
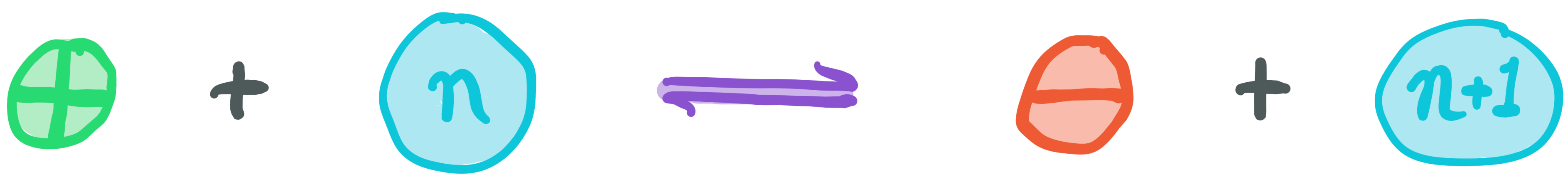


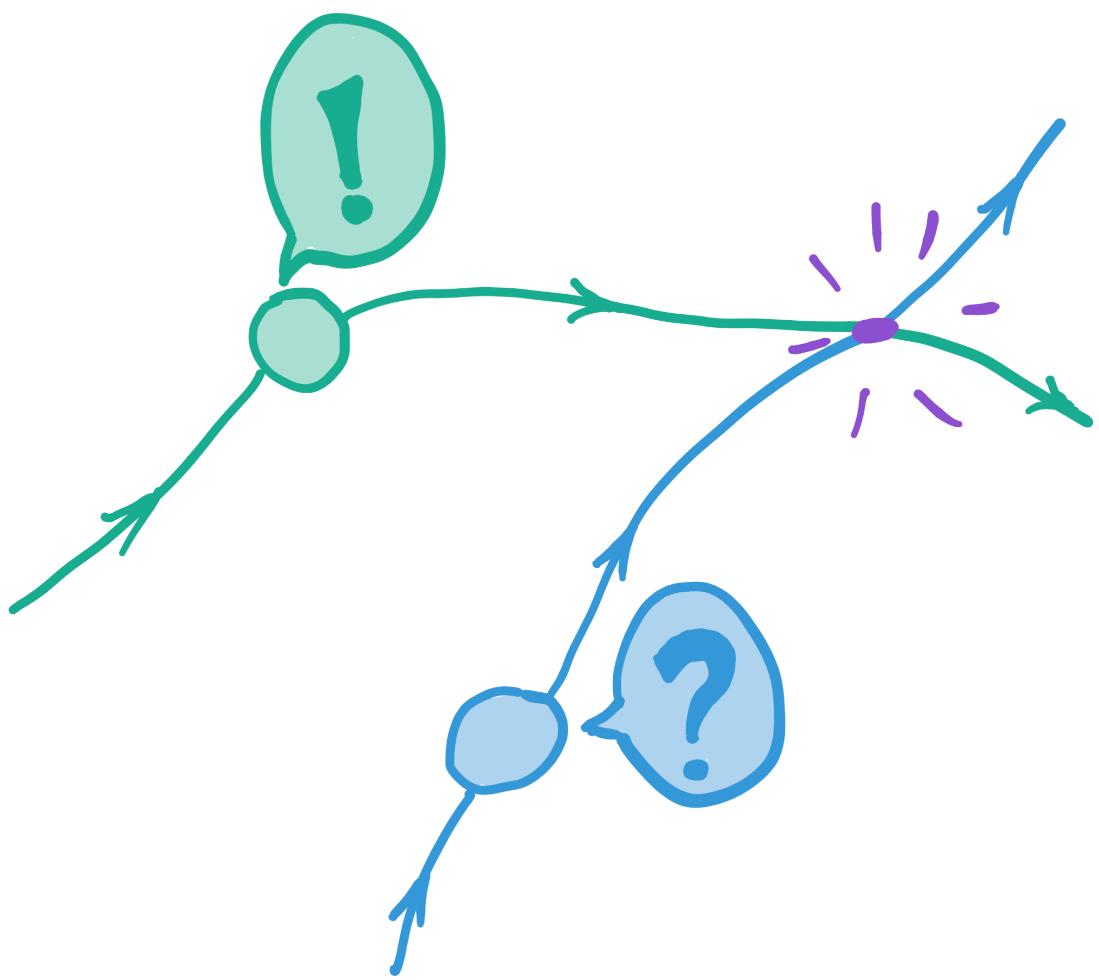
Async/Markovian RC

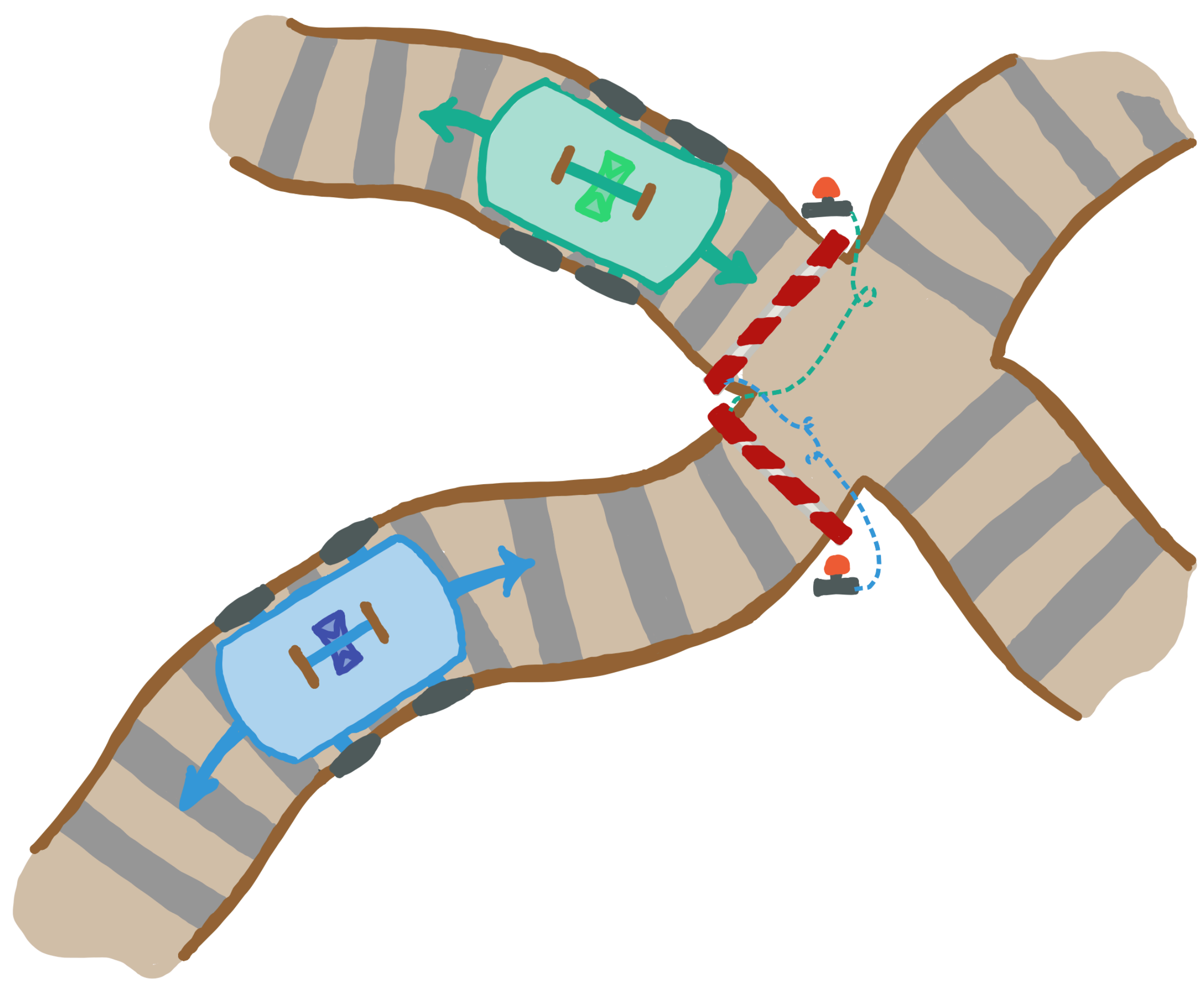


- realizes the adiabatic bound

$$\nu_C \lesssim r^{5/2} \sim V^{5/6} \qquad \nu_{C,1} \lesssim r^{-1/2} \sim V^{-1/6}$$

Asynchronous Sync





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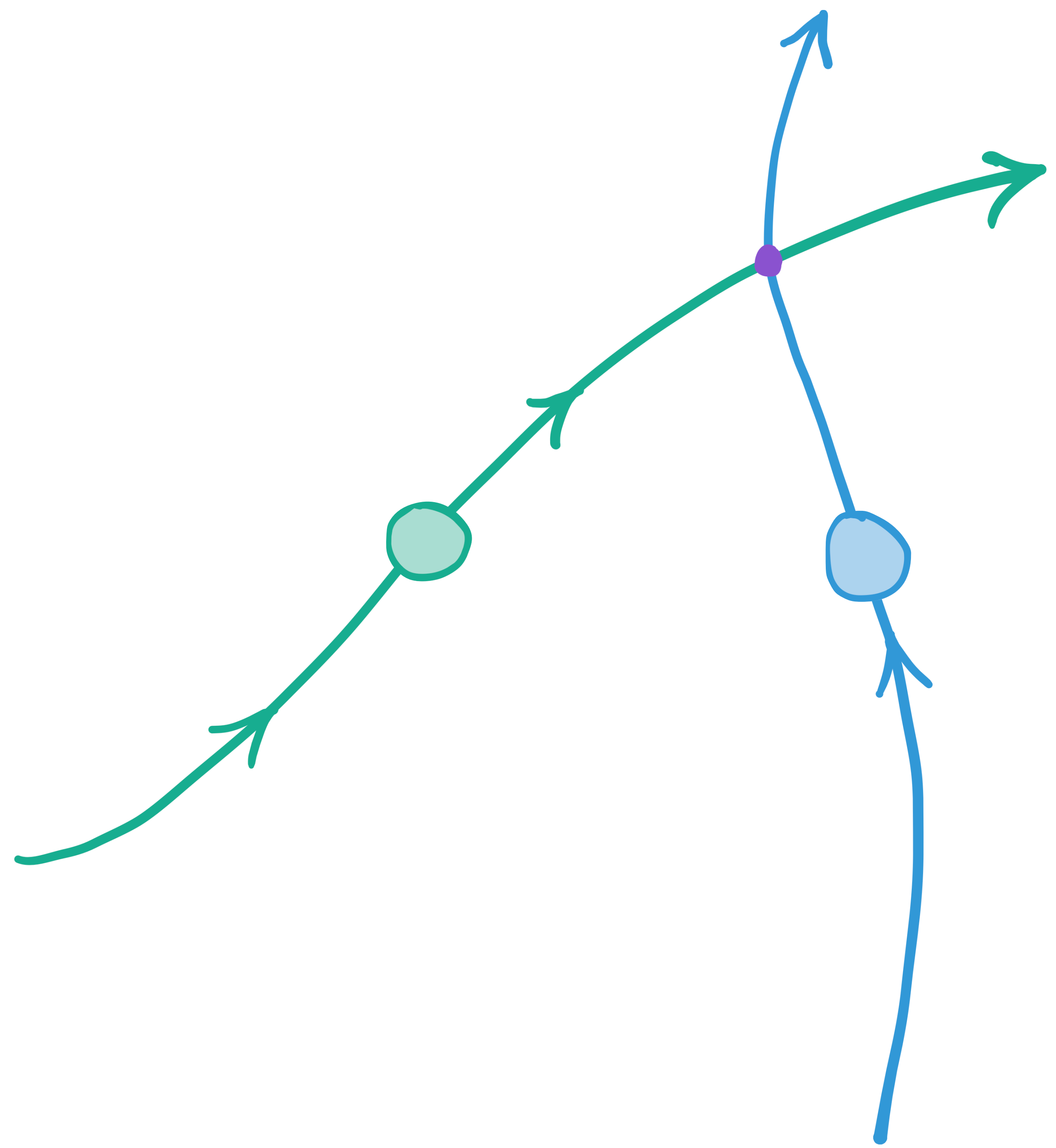

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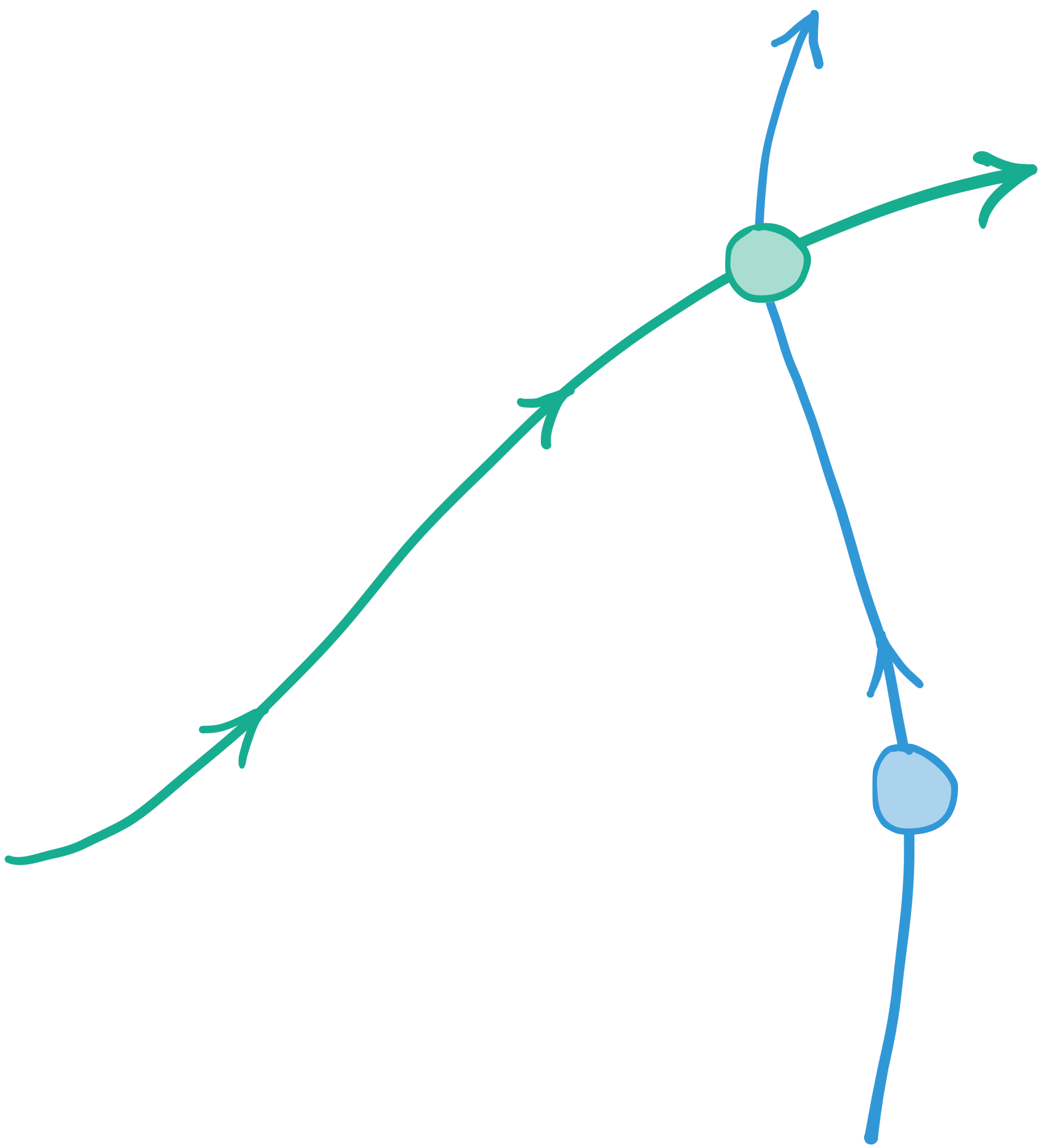

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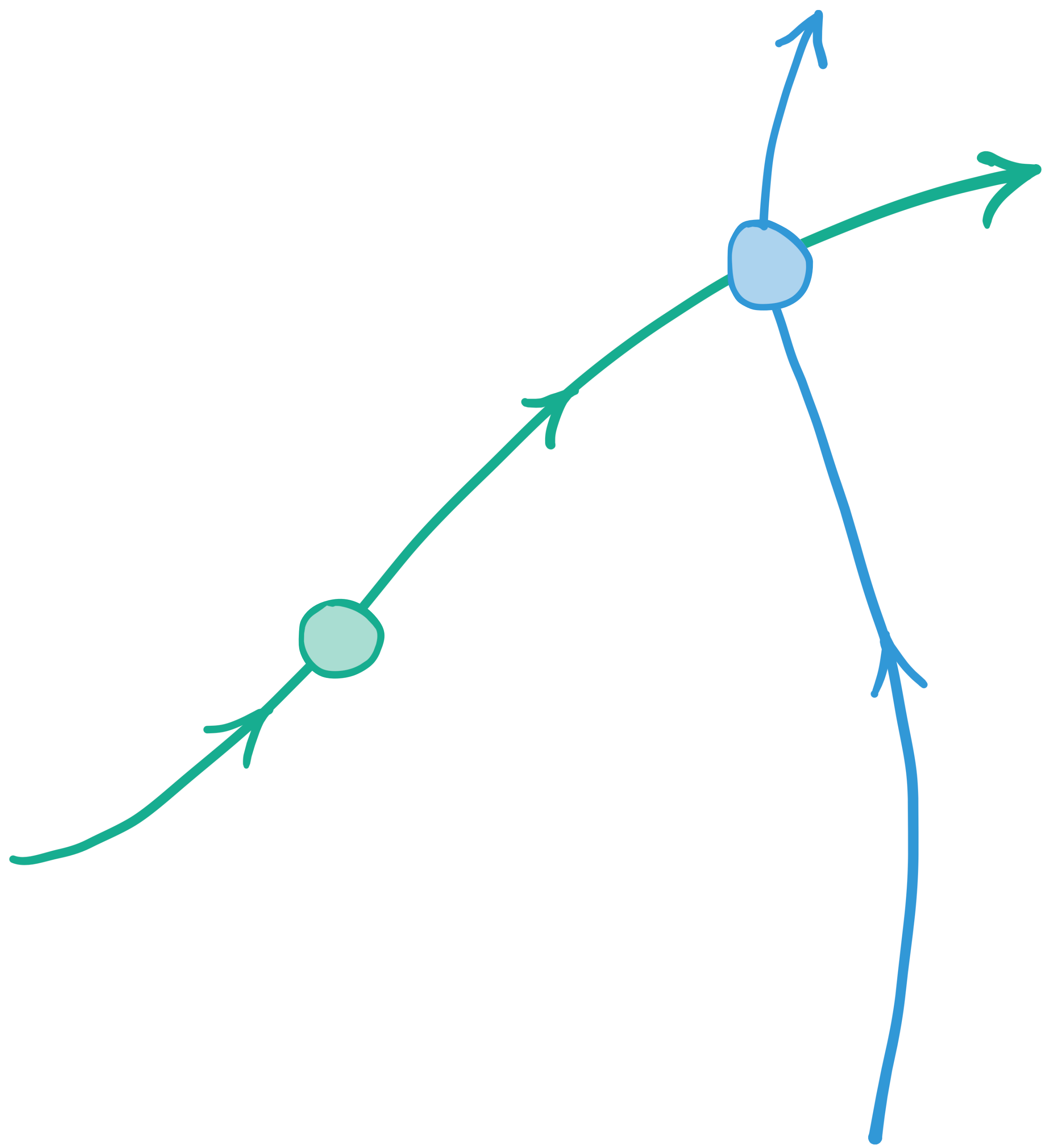

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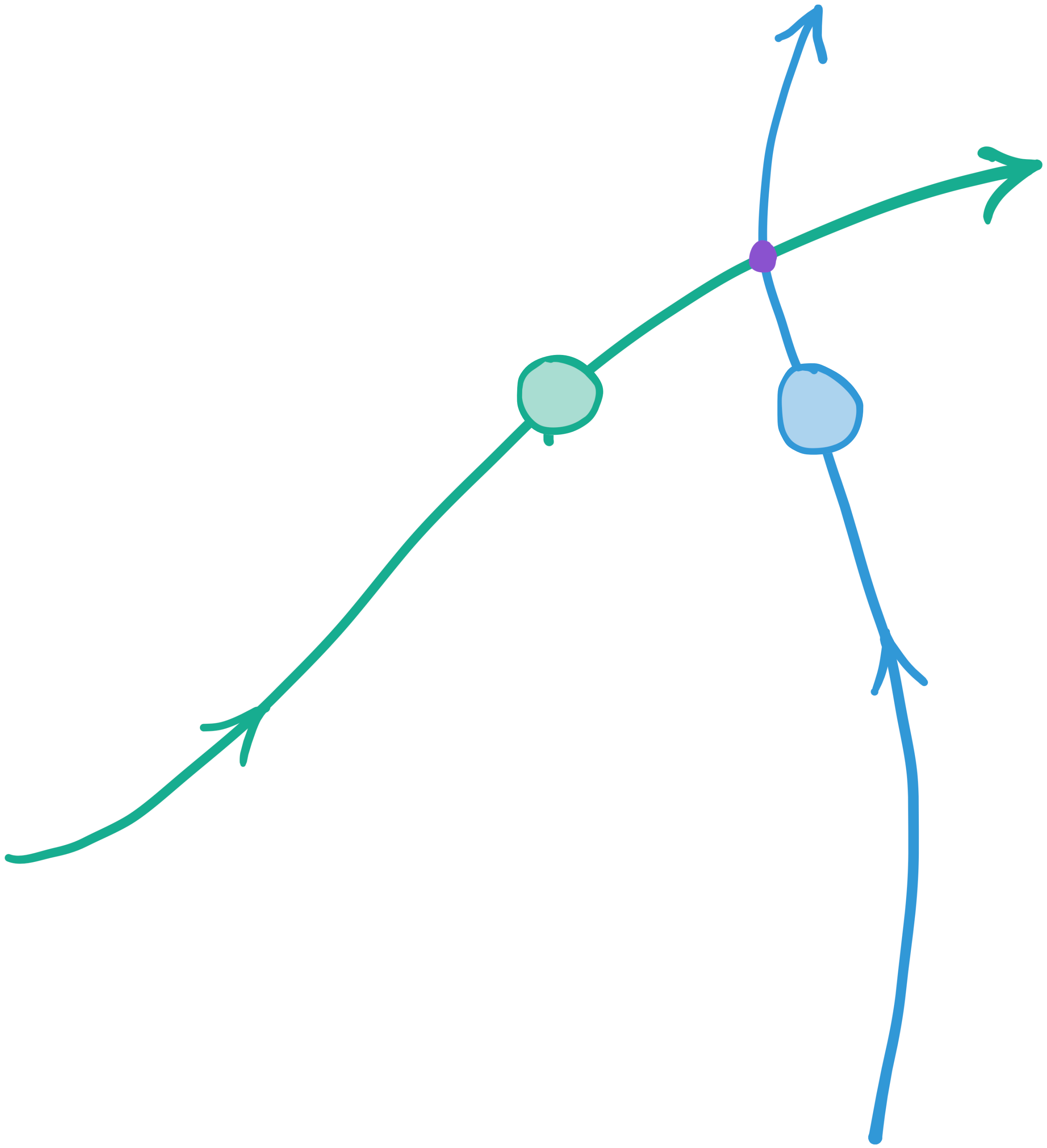

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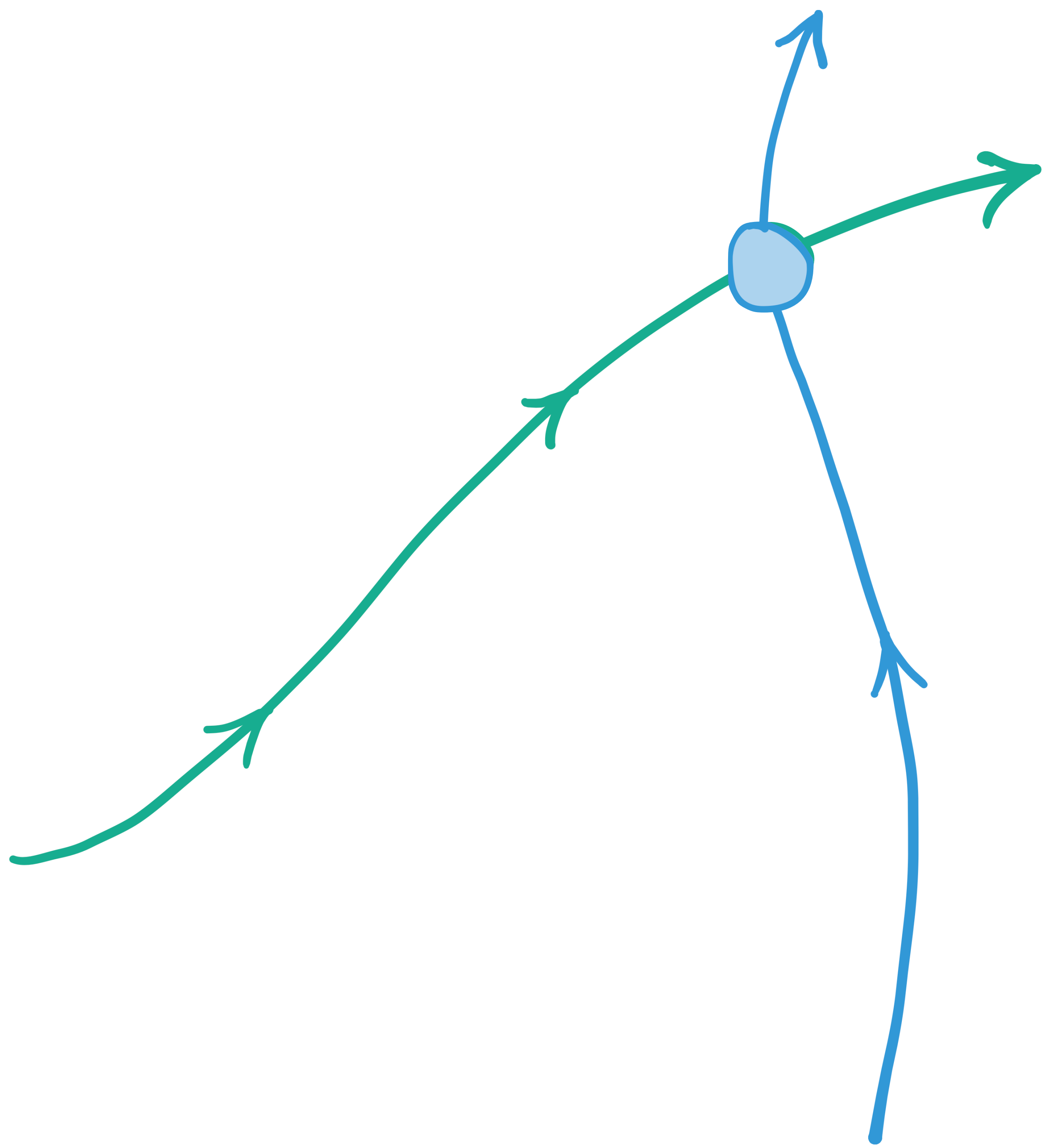

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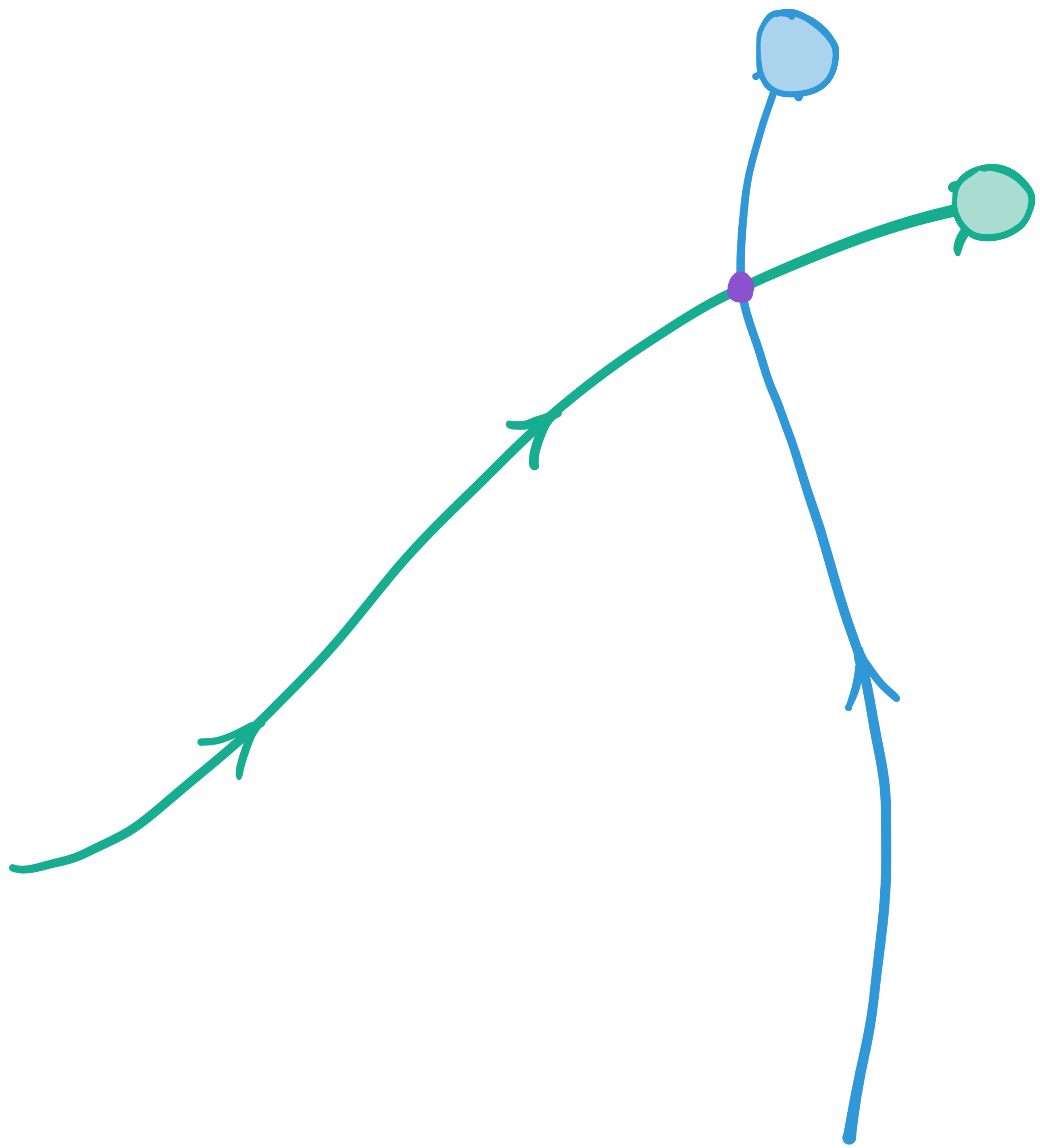

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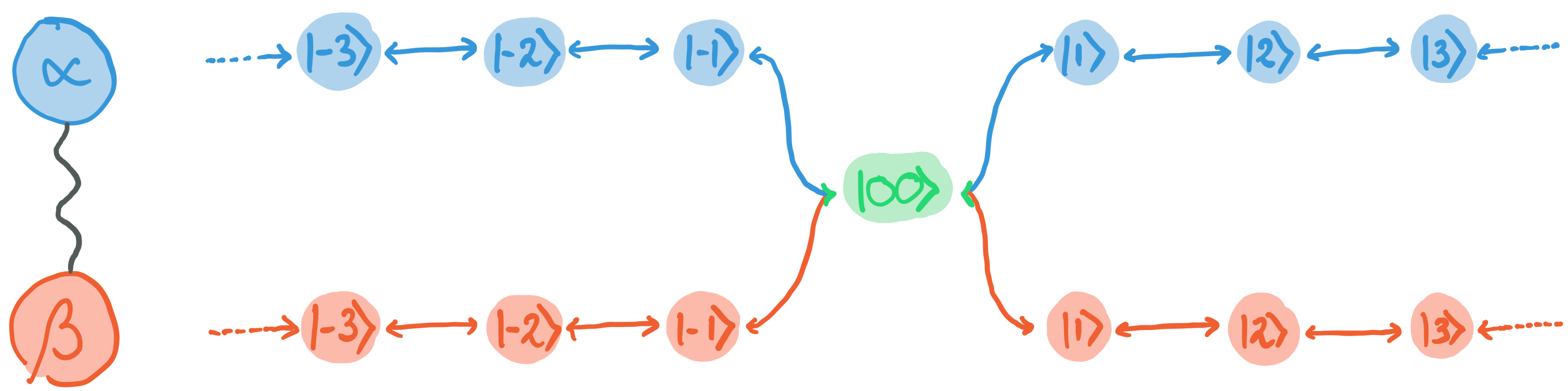

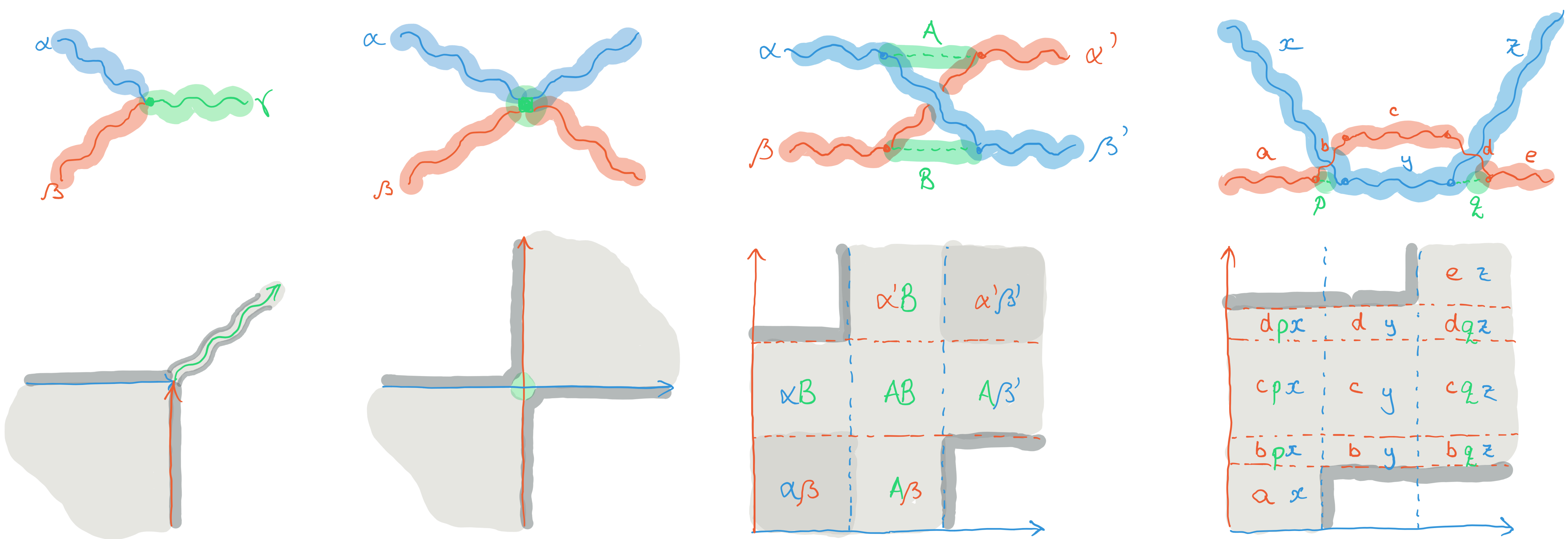

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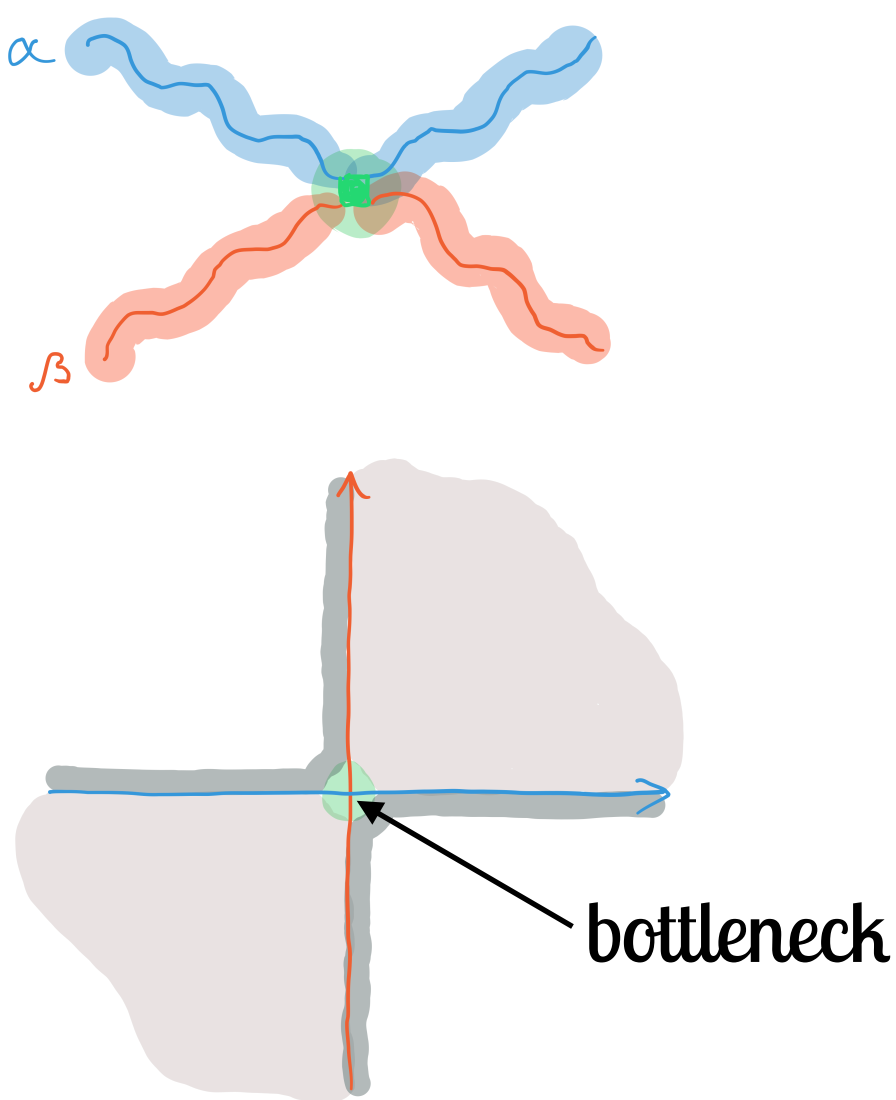

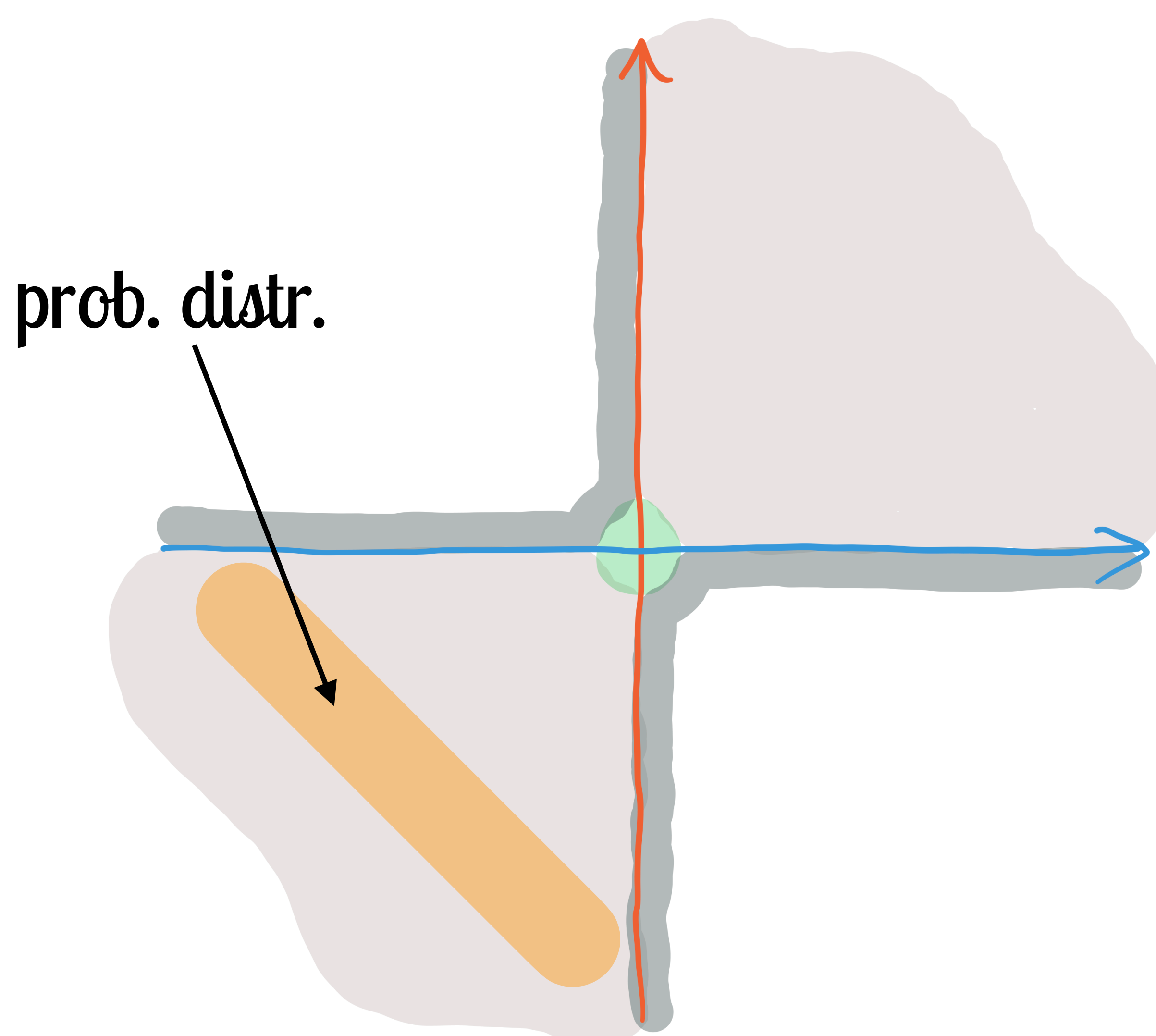

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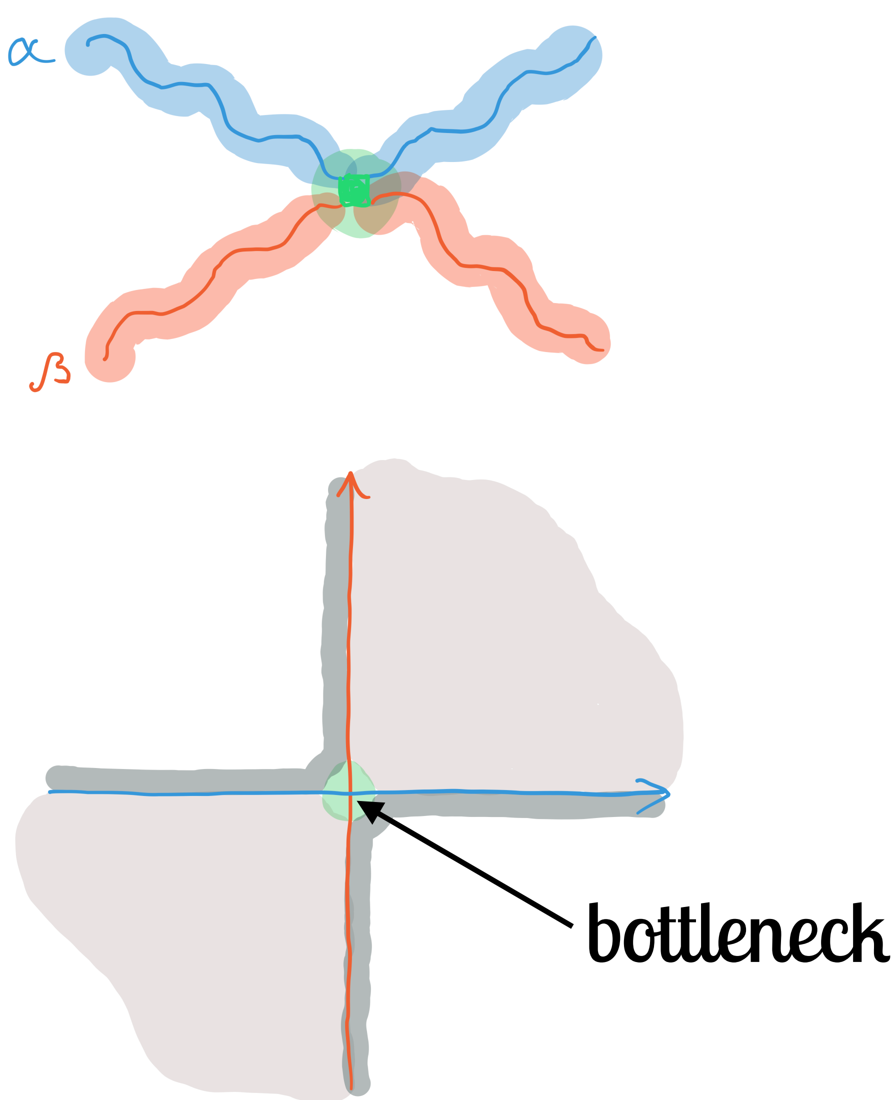

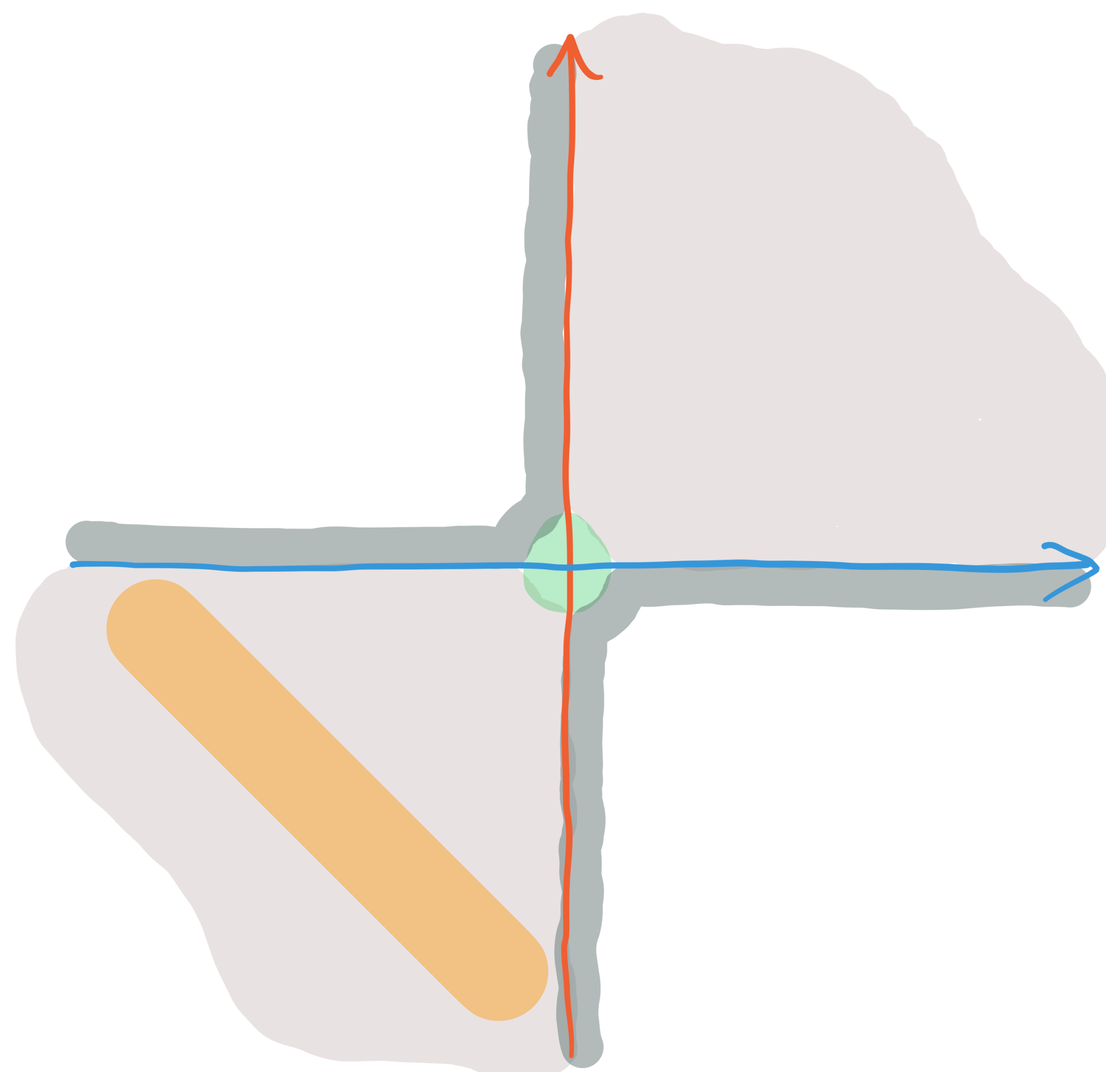

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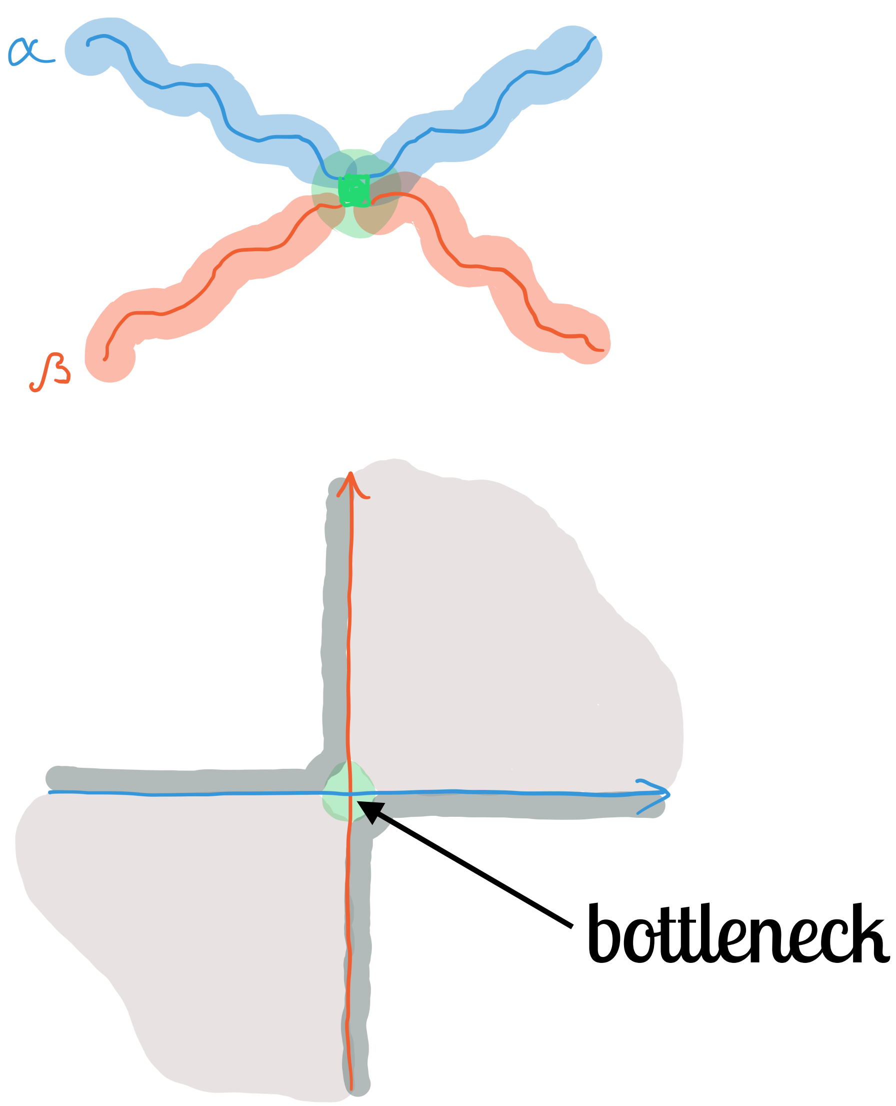

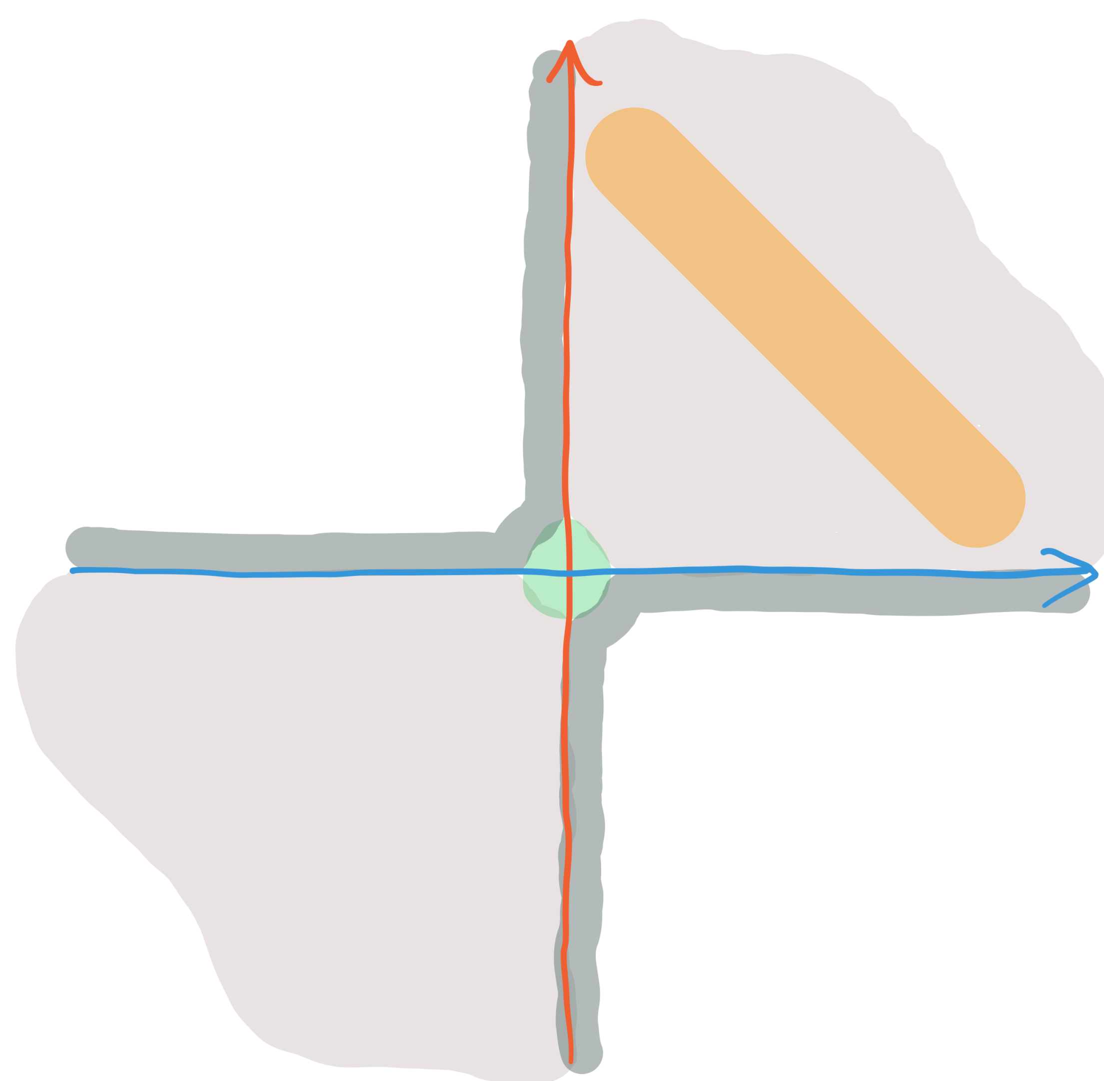

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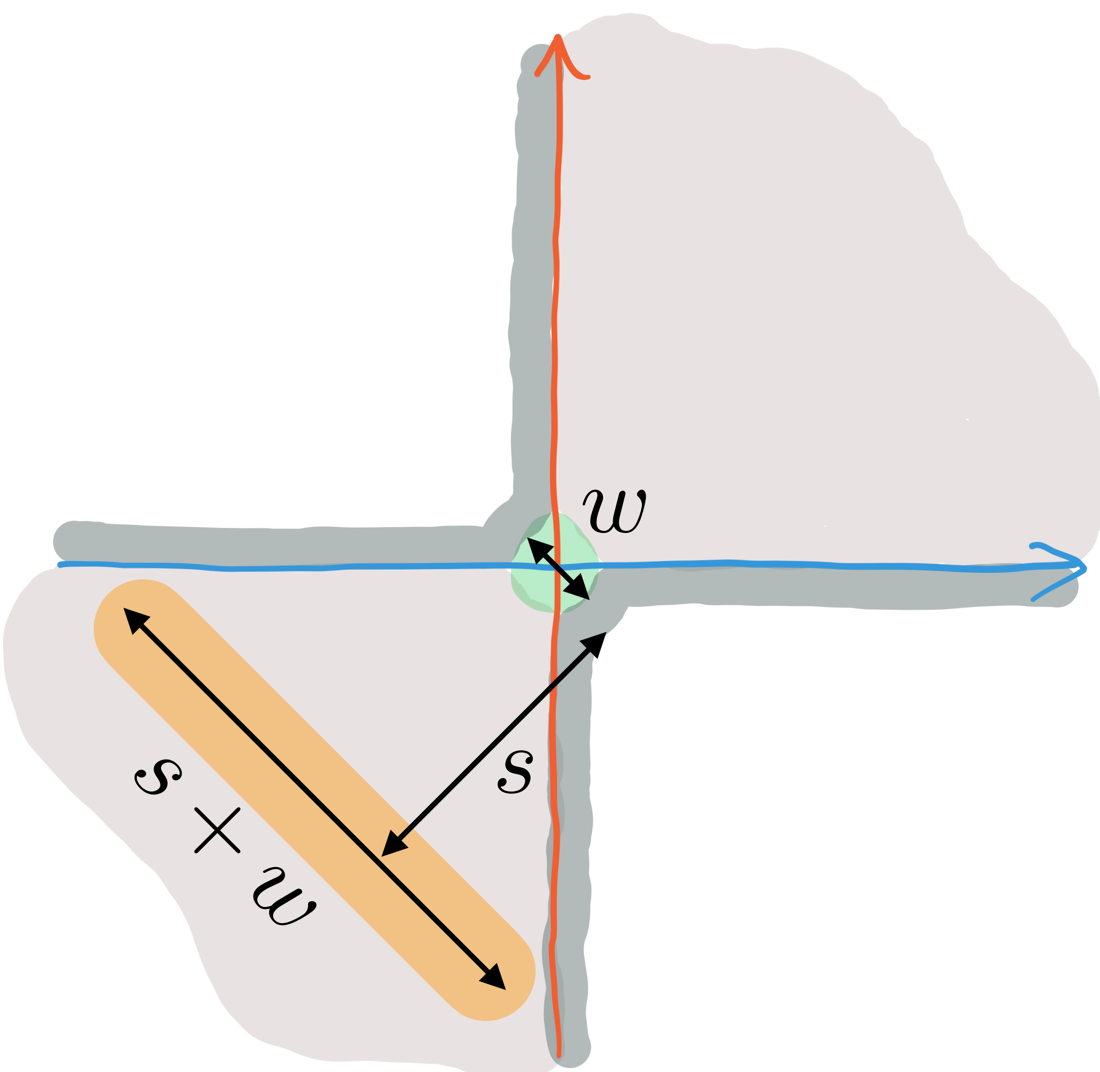


- info erasure model

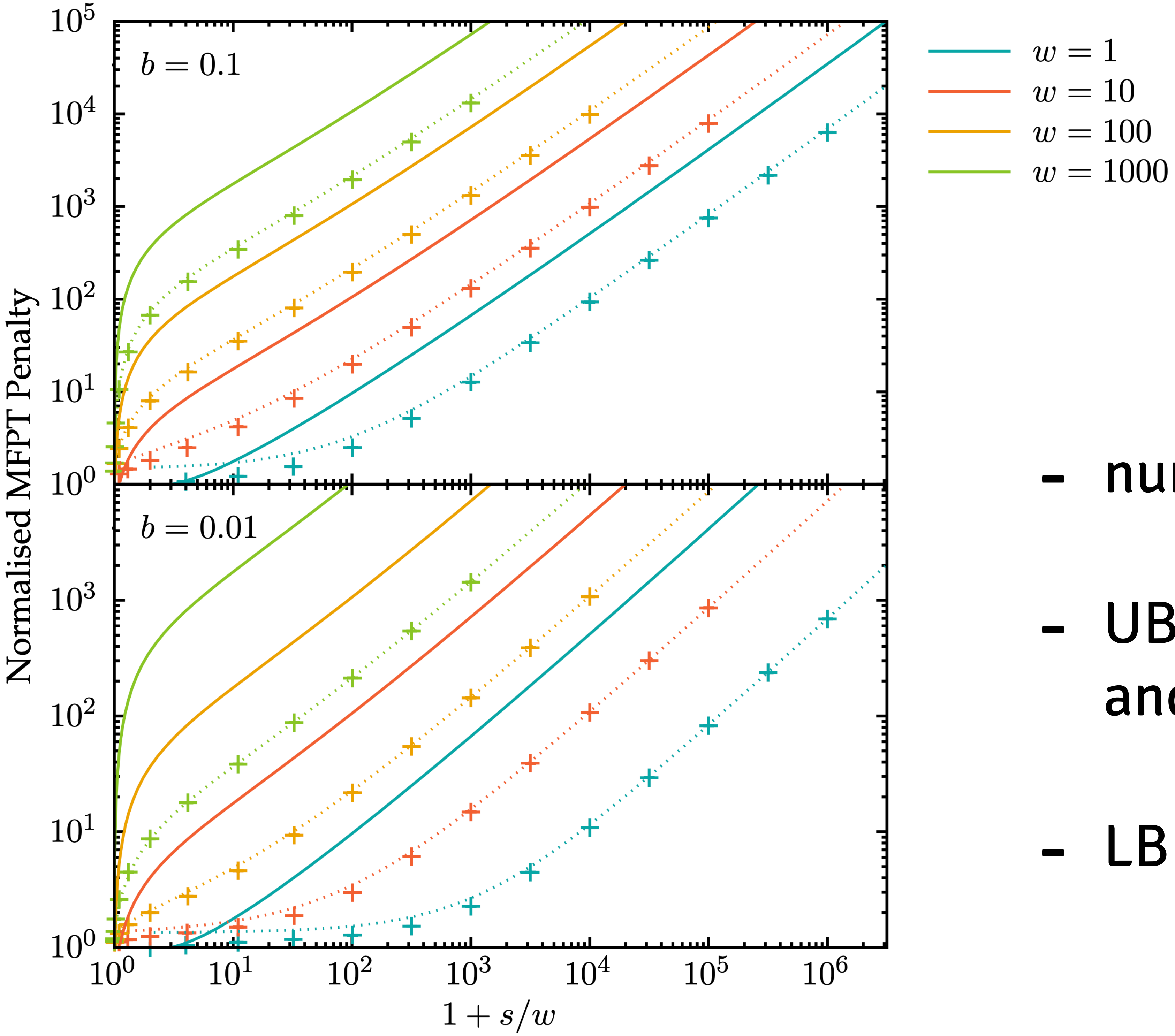
$$\Delta I \approx \log \frac{s+w}{w}$$

$$\Delta t \gtrsim \Delta I / \nu_{\text{irrev.},1}$$

$$\nu_{\text{irrev.},1} \sim r^{-1}$$

$$\nu_{\text{rev.},1} \sim r^{-1/2}$$

Asynchronous Sync



- numerical sim
- UB and LB from MFPT and discrete MC
- LB ~ info theory

Asynchronous Sync

- expensive, ~equivalent to entropy erasure
- significant parallelism/concurrency in async RC
  $\Rightarrow$ comp rate falls to ~irrev
- constrains algorithms to minimise sync or constrains net comp rate

Conclusions & Future Work

- RC > irrevC; adiabatic upper bound
  - for large & sustained comp
  - concrete const factors?
  - super-adiabatic special cases? (e.g. Pidaparthi+Lent)
- async sync expensive
  - sync sync performance?
    - preliminary results suggest cheap
    - algorithm design? mitigations for algorithmic desync?

Thank you!

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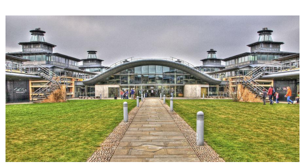
Vaire

MICKLEM LAB

UKRI Engineering and Physical Sciences Research Council



Department of Applied Mathematics and Theoretical Physics (DAMTP)

Hannah Earley — 2016–2021 — 'Modelling approaches to molecular computation' — EPSRC Project Reference 1781682
Hannah Earley — 2020/2022 — 'The $\aleph$-Calculus' — arXiv/Proceedings of RC22